MA 334 STUDY GUIDE FOR EXAM 2

Exam 2 will be on **Wednesday, November 1.** The exam will cover Sections 2.5, 2.6, 3.1, 3.2, as well as convergence in $\mathbb{R}^2$.
No books, notes, or calculators are allowed on the test.

You will be asked to
- State definitions and theorems.
- Answer true/false questions.
- Give proofs. These problems will be similar to homework problems.
  Make sure that you know how to solve all Hw problems.

Here is a list of definitions and theorems that you need to know.

Definitions.

- Upper bound, lower bound, supremum, and infimum.
- Subsequence.
- Limit point of a sequence.
- What does mean that a sequence $\{a_n\}$ in $\mathbb{R}^2$ converges to a?
- What does it mean that a set S in $\mathbb{R}^2$ is bounded?
- What does mean that a sequence $\{a_n\}$ in $\mathbb{R}^2$ is Cauchy?
- Continuity of f at c in Dom f (in terms of sequences and in terms of ε and δ).
- What does mean that $\lim_{x \to c} f(x) = L$? (in terms of sequences and in terms of ε and δ).
- What does mean that a function f is bounded on a set S?
- What does mean that a function f is uniformly continuous on a set $S \subset$ Dom f ?

Theorems.

- Any set $S \subset \mathbb{R}$ which is bounded above has a unique least upper bound.
- d is a limit point of a sequence $\{a_n\}$ if and only if there is a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ converging to d.
- Bolzano-Weierstrass Theorem (Theorem 2.6.2).
- Let b be an upper bound for a set S. Then $b = \sup S \Leftrightarrow$ there is a sequence of elements of S converging to $b \Leftrightarrow$ for any $\varepsilon > 0$ there is an element of S in $[b - \varepsilon, b]$.
- Let $a_n = (x_n, y_n)$ and $a = (x, y)$. Then $a_n \to a$ if and only if $x_n \to x$ and $y_n \to y$.
- A sequence $\{a_n\}$ in $\mathbb{R}^2$ cannot have more than one limit.
- A sequence in $\mathbb{R}^2$ converges if and only if it is a Cauchy sequence.
- Continuity of kf, $f \pm g$, fg, f/g (Theorem 3.1.1).
- Continuity of the composition (Theorem 3.1.2).
- A continuous function on a closed interval is bounded.
- A continuous function on a closed interval attains its maximum and minimum values.
- The Intermediate Value Theorem (Theorem 3.2.3).
- A continuous function on a closed interval is uniformly continuous.

True/False questions. You should determine whether or not a statement is true. If a statement is false, you must provide a counter example.

(1) If supremum of a set is finite, then the set itself is bounded.

(2) Infimum of a bounded set is finite.

(3) Supremum of a set always belongs to the set.

(4) For every set, the infimum is strictly less than the supremum.

(5) If $\{a_n\}$ and $\{b_n\}$ are bounded sequences, then $\sup\{a_n + b_n\} \geq \sup\{a_n\}$.

(6) If $\{a_n\}$ and $\{b_n\}$ are bounded sequences, then $\sup\{a_n + b_n\} = \sup\{a_n\} + \sup\{b_n\}$.

(7) Every sequence has at least one limit point.

(8) Every bounded sequence has at least one limit point.

(9) Every bounded sequence has a convergent subsequence.

(10) If a sequence has exactly one limit point, then it converges.

(11) If a sequence has a convergent subsequence, then it is bounded.

(12) If a sequence $\{a_n\} = \{(x_n, y_n)\}$ in $\mathbb{R}^2$ diverges, then both $\{x_n\}$ and $\{y_n\}$ diverge.

(13) If $\{x_n\}$ or $\{y_n\}$ diverges then the a sequence $\{a_n\} = \{(x_n, y_n)\}$ diverges.

(14) If $\{x_n\}$ has k limit points and $\{y_n\}$ has m limit points, then the sequence $\{a_n\} = \{(x_n, y_n)\}$ has km limit points.

(15) A sequence $\{a_n\} = \{(x_n, y_n)\}$ is bounded if and only if both $\{x_n\}$ and $\{y_n\}$ are bounded.

(16) If f and g are defined and continuous on S, then $\frac{f}{g}$ is also defined and continuous on S.

(17) If a continuous function is negative at a point of its domain, then it is negative on an open interval containing this point.

(18) Any continuous function on a closed interval is bounded.

(19) Any continuous function on a bounded interval is bounded.

(20) If f is defined on $[a, b]$, $f(a) = -1$, and $f(b) = 1$, then there is a point c in (a, b) such that $f(c) = 0$.

(21) If f is defined on $[a, b]$, $f(a) = 1$, and $f(b) = 1$, then there is no point c in (a, b) such that $f(c) = 0$.

(22) If f is a bounded continuous function on $[a, b]$, then the range of f is the interval $[m, M]$, where $m = \inf_{[a,b]} f$ and $M = \sup_{[a,b]} f$.

(23) If f is a bounded function on $[a, b]$, then the range of f is the interval $[m, M]$ as in (22).

(24) Any polynomial has at least one real zero.

(25) Any odd degree polynomial has at least one real zero.

(26) Any continuous function is uniformly continuous.

(27) Any uniformly continuous function is continuous.

(28) A continuous function on a closed interval $[a, b]$ is uniformly continuous.

(29) A continuous function on a bounded interval is uniformly continuous.