

MA 334 STUDY GUIDE FOR FINAL EXAM

Final Exam will be on **Friday, December 8, 6-8 p.m.** The exam will be cumulative, but the material of Sections 3.3, 3.5, 3.6, 4.1, and 4.2 will be emphasized. No books, notes, or calculators are allowed on the test.

You will be asked to

- State definitions and theorems.
- Answer true/false questions.
- Give proofs. You will be asked to prove some of the 6 theorems listed at the bottom of p.2, and there will be problems similar to homework problems from sections 3.3-4.2.

Here is a list of definitions and theorems that you need to know.

Definitions.

- What does mean that a sequence $\{a_n\}$ (in \mathbb{R}) converges to a ?
- What does mean that a sequence $\{a_n\}$ does not converge to a ?
- What does it mean that a sequence diverges to ∞ ? to $-\infty$?
- What does mean that a sequence is Cauchy?
- What are supremum and infimum of a set in \mathbb{R} ?
- What does it mean that a function f is continuous at a point c in $\text{Dom } f$ (in terms of sequences and in terms of ε and δ).
- What does mean that $\lim_{x \rightarrow c} f(x) = L$? (in terms of sequences and in terms of ε and δ).
- What does mean that $f(x)$ does not tend to L as $x \rightarrow c$?
- What does mean that a function f is uniformly continuous on its domain?
- For a bounded function f on $[a, b]$ and a partition P of $[a, b]$, what is the lower sum $L_P(f)$, the upper sum $U_P(f)$, and a Riemann sum $S_P(f)$?
- What does it mean that a bounded function f on $[a, b]$ is Riemann integrable? How $\int_a^b f(x) dx$ is defined?
- Let f be an unbounded continuous function on $(a, b]$. What does it mean that the improper integral $\int_a^b f(x) dx$ exists?
- Let f be a continuous function on $[a, \infty)$. What does it mean that the improper integral $\int_a^\infty f(x) dx$ exists?
- What does it mean that a function f is differentiable at a point $x \in \text{Dom } f$? How is $f'(x)$ defined?
- What does it mean that f is continuously differentiable on (a, b) ?

Theorems.

- A sequence (in \mathbb{R}) can have at most one limit.
- If a sequence converges, then it is bounded.
- A sequence of real numbers is a Cauchy sequence if and only if it converges to a (finite) real number.
- Every bounded monotone sequence converges.
- Bolzano-Weierstrass Theorem (Theorem 2.6.2).
- A continuous function on a bounded closed interval is bounded.
- A continuous function on a bounded closed interval attains its supremum and infimum.
- The Intermediate Value Theorem (Theorem 3.2.3).
- A continuous function on a bounded closed interval is uniformly continuous.

- Properties of upper and lower sums: Lemmas 1, 2, and 3 in Section 3.3.
- Any continuous function on $[a, b]$ is Riemann integrable.
- Convergence of Riemann sums to the integral for a continuous function (Corollary 3.2.2).
- Properties of Riemann integral: Theorems 3.3.3, 3.3.4, and 3.3.5.
- Any monotone bounded function on $[a, b]$ is Riemann integrable.
- If a bounded function on $[a, b]$ is continuous except for finitely many points, then it is Riemann integrable.
- If f differentiable at x , then f is continuous at x .
- Differentiation rules: Theorems 4.1.2 and 4.1.3.
- Suppose that f is continuous on $[a, b]$ and it attains its maximum or minimum value at $c \in (a, b)$. If f is differentiable at c , then $f'(c) = 0$.
- Rolle's Theorem.
- The Mean Value Theorem.
- The Fundamental Theorem of Calculus, Parts I and II.

In addition, you should be able to **prove** the following theorems:

- A sequence (in \mathbb{R}) can have at most one limit.
- If a sequence converges, then it is bounded.
- A continuous function on a closed interval is bounded.
- Product rule for the derivative (p.124).
- If f differentiable at x , then f is continuous at x .
- The Fundamental Theorem of Calculus, Part I.

True/False questions. You should determine whether or not a statement is true. If a statement is false, you must provide a counter example.

Also, go over the true/false questions from Exam 1 and 2 study guides.

- (1) Every bounded function on $[a, b]$ is Riemann integrable.
- (2) Every continuous function f on $[a, b]$ is Riemann integrable.
- (3) If f is Riemann integrable on $[a, b]$, then f is continuous on $[a, b]$.
- (4) If a bounded function on $[a, b]$ is continuous except for at finitely many points, then it is Riemann integrable.
- (5) If a function f on $[a, b]$ is Riemann integrable then it is continuous except for at finitely many points.
- (6) For any continuous function f on $[a, b]$ and any two partitions P and Q of $[a, b]$,
$$L_P(f) \leq \int_a^b f(x) dx \leq U_Q(f).$$
- (7) For any continuous function f on $[a, b]$, partition P , and Riemann sum $S_P(f)$,
$$L_P(f) \leq S_P(f) \leq U_P(f).$$
- (8) Let f be a bounded function on $[a, b]$. If for any $n \in \mathbb{N}$ there is a partition P such that
$$U_P(f) - L_P(f) \leq \frac{1}{n}$$
 then f is Riemann integrable.
- (9) Let f be a bounded function on $[a, b]$. If $L_P(f) < U_Q(f)$ for any partitions P and Q , then f is not Riemann integrable.
- (10) Let f be a bounded function on $[a, b]$. If there is $\varepsilon > 0$ such that $L_P(f) < U_Q(f) - \varepsilon$ for any partitions P and Q , then f is not Riemann integrable.
- (11) Let f be a nonnegative bounded function on $[a, b]$. If for any $\varepsilon > 0$ there is a partition P such that $U_P(f) \leq \varepsilon$ then f is Riemann integrable.
- (12) If f is continuous on $[a, b]$, then $m(b - a) \leq \int_a^b f(x) dx \leq M(b - a)$, where $m = \inf_{[a,b]} f$ and $M = \sup_{[a,b]} f$.
- (13) If f is a continuous function on $[a, b]$, then $|\int_a^b f(x) dx| \leq \int_a^b |f(x)| dx$.
- (14) If f is continuous and bounded on (a, b) then $\int_a^b f(x) dx$ exists.
- (15) If f is unbounded on (a, b) then the Riemann integral does not exist (in the sense of the supremum of lower sums = infimum of the upper sums).
- (16) If f is unbounded on (a, b) then the improper integral $\int_a^b f(x) dx$ does not exist.
- (17) If f is a non-negative continuous unbounded function on $[0, \infty)$, then the improper integral $\int_0^\infty f(x) dx$ does not exist.
- (18) If f is continuous at a , then f is differentiable at a .
- (19) If f is differentiable at a , then f is continuous at a .
- (20) If f is differentiable and bounded on (a, b) , then f' is bounded on (a, b) .

- (21) If a continuous function f attains its maximum at c , then $f'(c) = 0$ or $f'(c)$ does not exist.
- (22) If f is a continuous function on $[a, b]$ such that $f(a) = f(b)$ then there is a point $c \in (a, b)$ such that $f'(c) = 0$.
- (23) If f is a continuously differentiable function on $[0, 2]$, $f(0) = 1$, and $f(2) = 7$ then there exists a point $c \in (0, 2)$ such that $f'(c) = 3$.
- (24) If f is a continuously differentiable function on $[0, 2]$ and $f'(x) \leq 3$ for all x in $[0, 2]$, then $f(x) \leq 3x$ for all x in $[0, 2]$.
- (25) If $f(a) \leq g(a)$ and $f'(x) \leq g'(x)$ for all $x \in [a, b]$, then $f(x) \leq g(x)$ for all $x \in [a, b]$.
- (26) The function $F(x) = \int_1^x t \sin \frac{1}{t} dt$ is continuously differentiable on $[1, \infty)$.
- (27) For any continuous function f on $[a, b]$ there exists a differentiable function g on $[a, b]$ such that $f = g'$.