

Math 334 Homework 7

due Wednesday, 10/18

- Section 2.6 Problems 3, 4, 6.
- Let $a_n = (x_n, y_n)$ and $a = (x, y)$.
Prove that $a_n \rightarrow a$ if and only if $x_n \rightarrow x$ and $y_n \rightarrow y$.
- Prove that a sequence $\{a_n\}$ in \mathbb{R}^2 cannot have more than one limit.
- Let $a_n = \left((-1)^n + \frac{1}{n}, \sin \frac{n\pi}{2} \right)$. Find all limit points of the sequence $\{a_n\}$.
- Let $\{a_n\} = \{(x_n, y_n)\}$ be a sequence in \mathbb{R}^2 .
Suppose that x is a limit point of $\{x_n\}$ and y is a limit point of $\{y_n\}$.
Does it follow that (x, y) is a limit point of $\{a_n\}$?
Prove it or give a counterexample.
- Consider the space $C([0, 1])$ of continuous functions on $[0, 1]$ with the norm
 $\|f\| = \max_{x \in [0, 1]} |f(x)|$.
Let f_0 be the zero function, i.e. $f_0(x) = 0$ for all x in $[0, 1]$.
Find a sequence of non-constant functions $\{f_n\}$ such that $f_n \rightarrow f_0$.