

MA 490/590 Review Sheet for the Final Exam

Midterm Exam will be on **Wednesday, April 30, 10:30 a.m. - 12:30 p.m.**

No books or notes will be allowed on the test.

You need to know the main definitions and statements, and to be able to solve problems similar to homework problems.

Here is a list of topics that you need to review for the test:

- **Strictly increasing continuous functions of an interval:**
 - Find fixed points, and tell which ones are attracting / repelling from the graph or from the derivatives;
 - Tell what $f^n(x)$ tends to for a given x .

- **Contractions:**
 - Definition;
 - Statement of the Contraction Principle;
 - How to tell if a real differentiable function is a contraction and how to find λ .

- **Circle rotations R_α :**
 - Definition; Inverse;
 - Description of the orbits in the case of rational α ;
 - Density of orbits in the case of irrational α .

- **Linear maps of the plane:**
 - Which "model" matrices a 2 by 2 real matrix can be similar to;
 - For each of the model matrices: the action on the unit square, and a description of orbits of points;
 - Eventually contracting maps: definition, and its implication;
 - If B is contracting then $A = CBC^{-1}$ is eventually contracting.

- **Linear maps of the torus \mathbb{T}^2 :**
 - Which matrices define an invertible map of \mathbb{T}^2 ;
 - For a hyperbolic linear map f of \mathbb{T}^2 :
 - The eigendirections are irrational;
 - A point is periodic if and only if its coordinates are rational, and hence periodic points are dense;
 - f is mixing.

- For a **map** $f : X \rightarrow X$, you need to know the definitions of
 - A fixed point;
 - A periodic point;
 - A period and the prime period of a periodic point.

- For a **homeomorphism** $f : X \rightarrow X$, you need to know
 - The definition;
 - The definition of an invariant set;
 - What are the orbit, positive semiorbit, and negative semiorbit of x ;
 - What does it mean that f is topologically transitive;
 - A criterion for topological transitivity (4 equivalent conditions).
 - What does it mean that f is minimal, and the equivalent statement about closed invariant sets.

For a continuous map f :

 - Topological mixing: definition, and that it implies transitivity;
 - Chaos;
 - Sensitive dependence on initial conditions;
 - Chaos implies sensitive dependence, but the converse is not true.

- **Coding:**
 - General description;
 - Coding for E_2 based on intervals $[0, 1/2]$ and $[1/2, 1]$;
 - Implications of coding for E_2 : binary expansion of a point with dense orbit; periodic points, non-recurrent points; invariant sets.

- **Topological entropy:**
 - Definition;
 - Entropy is zero for contractions and isometries;
 - Entropy of E_m is $\ln m$.