

MA 490/590 Homework 1

due Wednesday, 1/16

Exercises **1.2.3** (What is the number a called?);

1.2.17 (Correction: it should be 1.65 **billion**, not **million**);

1.3.1;

and the following problems:

1. Suppose that in example 1.2.3, $a_0 = 12$ tons and $a_1 = 15$ tons.

Find $|a_n - a_{n-1}|$ for $n \geq 2$.

2. Use Newton's method to approximate the zero of $f(x) = x^3 - 5$.

Take $x_0 = 2$ and find x_1, x_2 . Give the answers as fractions, not decimals.

Use a calculator to find the error $x_n - \sqrt[3]{5}$ for $n = 0, 1, 2$.

3. In the example about populations of two planets, suppose that initially 30% of the total population resides on planet X and 70% on planet Y , i.e. $x_0 = .3$ and $y_0 = .7$, and for $n \geq 0$

$$x_{n+1} = .95x_n + .1y_n$$

$$y_{n+1} = .05x_n + .9y_n$$

(a) Find x_1 and y_1 , then x_2 and y_2 .

(b) Assuming that $x_n \rightarrow x$ and $y_n \rightarrow y$ as $n \rightarrow \infty$, find x and y .

Does the answer depend on x_0 and y_0 ?

4. For the differential equation

$$\frac{dP}{dt} = P(1 - P) \left(1 - \frac{P}{2}\right), \quad t \geq 0,$$

find where the slope $\frac{dP}{dt}$ is positive and where it is negative. Then draw the slope field and sketch several solutions. Describe the behavior of the solutions depending on the initial value P_0 .