

# MA 490/590 REVIEW SHEET FOR MIDTERM EXAM

**Midterm Exam** will be on **Monday, March 17**.

No books or notes will be allowed on the test.

You need to know the main definitions and statements, and to be able to solve problems similar to homework problems.

Here is a list of topics that you need to review for the test:

- **Increasing continuous functions of an interval:**

For a given graph, you should be able to

- Find fixed points and tell which ones are attracting / repelling;
- Tell what  $f^n(x)$  tends to for a given  $x$ ;
- Give examples of invariant sets in the case of an invertible function.
- Also, for a fixed point  $x$  of a differentiable function  $f$ , you should know what  $f'(x)$  tells about  $x$  being attracting / repelling / superattracting.

- **Contractions:**

- Definition;
- Statement of the Contraction Principle;
- How to tell if a real differentiable function is a contraction and how to find  $\lambda$ ;
- Same questions for  $f(x, y) = (ax + b, cy + d)$ ;
- How to find the fixed point;
- How to estimate the distance between  $f^n(x)$  and the fixed point.

- **Fractals:**

The construction of a fractal in the plane will be described. You should be able to

- Find the area at the  $n^{\text{th}}$  step;
- Find the total perimeter at the  $n^{\text{th}}$  step;
- Find the fractal dimension;
- Find the box dimension.

- **Circle rotations  $R_\alpha$ :**

- Definition;
- Inverse;
- Description of the orbits in the case of rational  $\alpha$ ;
- Density of orbits in the case of irrational  $\alpha$ , Prop. 4.1.1.  
(In this case  $\mathbb{R}_\alpha$  is minimal and hence transitive.)
- Uniform distribution, Prop. 4.7.1.

- **Linear flow on the torus  $\mathbb{T}^2$ :**

- Definition;
- How to find the slope from the formula;
- What the orbits look like in the unit square;
- For which slopes the orbits are closed, i.e. all points are periodic;
- For which slopes the orbits are dense.

- For a **map**  $f : X \rightarrow X$ , you need to know the definitions of

- A fixed point;
- A periodic point;
- A period and the prime period of a periodic point.

- For a **homeomorphism**  $f : X \rightarrow X$ , you need to know

- The definition;
- The definition of an invariant set;
- What are the orbit, positive semiorbit, and negative semiorbit of  $x$ ;
- What does it mean that  $f$  is topologically transitive;
- What does it mean that  $f$  is minimal, and the equivalent statement about closed invariant sets.