

Math 507 Exam 2 – Review Problems

The exam covers Sections 3.7, 3.9, 4.2, 4.3.

1. Find the general solution of the equation $y'' + y' - 2y = x^3 + e^x$ using the method of undetermined coefficients.

2. Find the general solution of the equation $y'' + 4y' + 4y = 2e^{-2x}$ using the method of variation of parameters.

3. Find the general solution of the equation. To find a particular solution, you can use any method (including "guess and check").

(a) $y'' - 2y = x + 1$

(b) $y'' + 2y' + 5y = x$

(c) $x^2y'' + 5xy' + 4y = 8, \quad x > 0$

(d) $x^2y'' + xy' - y = \frac{1}{x}, \quad x > 0$

4. Find the general solution of the system of differential equations. If a system is inconsistent or redundant, show it, and find all solutions in the redundant case.

(a) $x' = 1 - y$
 $y' = 3 - 9x$

(b) $x' + x + y' = e^t$
 $x'' - x + y'' - y' = 3t$

(c) $x'' = x - 2y$
 $y'' = 2x - 4y$

(d) $x' = -y$
 $x'' - x' = y - y'$

5. Find the general solution of the equation as a power series about the point $x = x_0$: obtain the first 4 non-vanishing terms in the power series for $y_1(x)$ and $y_2(x)$, verify that y_1 and y_2 are LI, and find the radius and the interval of convergence of the solutions.

(a) $(1 + x^2)y'' - y = 0, \quad x_0 = 0$

(b) $xy'' - 2y' + xy = 0, \quad x_0 = 1$

6. Identify all singular points for the equation $(x + 1)^2y'' + (x + 1) \ln |x| \cdot y' + xy = 0$ and classify each one as regular or irregular.

7. Verify that $x = 0$ is a regular singular point and use the method of Frobenius to obtain the general solution of the equation. On what interval the solution is valid?

(a) $x^2y'' + x(1 + x)y' - \frac{1}{4}y = 0, \quad x > 0$

(b) $xy'' + y' - xy = 0, \quad x > 0$