Math 507 Final Exam – Review Problems

The final exam will be on Wednesday, May 5, 6-8 p.m. in ILB 370.

The exam will be cumulative, but the material of Chapters 5 and 7 will be emphasized. A list of Laplace transforms will be provided on the exam. Note that there will be a problem on the method of Frobenius.

In problems 1-9 find the general solution of the equation. If an initial condition is given, find the particular solution and state the interval of validity of this solution. You may use any method covered in this course.

1. $y' = \frac{1}{x^2y^2}, \ y(1) = 2$

2. $(e^x + y)\,dx - (\sin y - x)\,dy = 0, \ y(0) = 0$

3. $y'' + 2y' = x^2 + \sin x$

4. $y' + y = x$

5. $x^2y'' - xy' - 3y = 4x$

6. $y' - xy = 0$

7. $y^{(4)} + 2y''' + y'' = 0$

8. $xy' + y = 6x^2, \ y(-2) = 8$

9. $x^2y'' + 3xy' + y = 0$

10. Find the general solution of the system of differential equations.

(a) $x' = x - 8y + 1$
$y' = -x - y - 3t^2$

(b) $x'' + x - 2y = 0$
$y'' + 2x - 4y = 0$

11. Find the general solution of the equation as a power series about the point $x = 0$: obtain the first 4 non-vanishing terms in the power series for $y_1(x)$ and $y_2(x)$, verify that y_1 and y_2 are LI, and find the radius and the interval of convergence of the solutions.

$(2 - x)y'' + xy' + y = 0.$

12. Verify that $x = 0$ is a regular singular point and use the method of Frobenius to obtain the general solution of the equation. On what interval is the solution valid?

(a) $3xy'' + y' + y = 0$

(b) $xy'' + 2y' + (1 + x)y = 0$

13. Find the transforms (if they exist)

(a) $L\{t^4 e^{3t}\}$

(b) $L\{H(t-2)\cosh(3(t-2))\}$

(c) $L\{\frac{\cos(2t)}{t}\}$

(d) $L\{f(t)\}$, where $f(t) = \frac{t}{2}$, $0 \le t < 2$, and 2 is the minimal period of $f(t)$.

(e) $L^{-1}\{\frac{1}{s^2-3s+2}\}$

(f) $L^{-1}\{\frac{1}{s^2+2s+10}\}$

(g) $f(0)$, where $f(t) = L^{-1}\{\frac{2s}{(s^2+1)^2} \cdot \frac{1}{s-3} - 2\frac{1}{s-3}\}$

14. Solve the following equations using Laplace transform

(a) $x'' - x = H(t-3)$

(b) $x' - x = f(t)$, where $f(t) = \begin{cases} 0, & t < 1 \\ 2, & 1 \le t < 4 \\ 0, & t > 4 \end{cases}$

(c) $3x' + x = e^{2t}$

15. Determine the equation of phase trajectories and sketch enough phase trajectories to show the essential features of the phase portrait. Use arrows to indicate the direction of movement along those trajectories.
Find all equilibrium points (i.e. the points where $x' = y' = 0$).

(a) $x' = -xy, \quad y' = 4x^2$

(b) $x' = 3xy, \quad y' = -xy$

(c) $x' = 2y, \quad y' = x$