MA 508 Comprehensive Exam – Review

The comprehensive exam will cover the following topics. For each topic, review exercises are listed. All problems from the book were assigned as Hw problems in MA 508.
This test will be closed book. You may use up to three standard sheets of notes.

Fourier series. A periodic function f is given over one period as follows
(a) $f(x) = 2$ on $(0, 4]$, and -2 on $(4, 8]$
(b) $f(x) = 0$ on $(-4, -2]$, -1 on $(-2, 2]$, and 0 on $(2, 4]$
(c) $f(x) = 0$ on $[0, 2\pi)$, and 3 on $[2\pi, 4\pi)$

Work out the Fourier series of f. At which values of x does the series fail to converge to $f(x)$? Describe all such values. To what values does it converge at those points?

Before calculating the series sketch the graph of f on a reasonably large interval and find out if f is even, odd or neither. For example in (b) f is even, and it is clear from the graph that its Fourier series fails to converge to $f(x)$ at $x = 2+4k$, where k is an integer.

Fourier transform. 17.10 **6**(b,e,f,h,k,l)
If there is such a problem on the exam, a list of Fourier transforms will be provided.

Diffusion equation.
Find the steady-state solution of the problem
(a) $\alpha^2 u_{xx} = u_t,\ \ u(0,t) - 2u_x(0,t) = 3,\ \ u_x(3,t) = 2,\ \ u(x,0) = 5.$
(b) $\alpha^2 u_{xx} = u_t,\ \ u(0,t) + u_x(0,t) = 3,\ \ u(2,t) + u_x(2,t) = 1,\ \ u(x,0) = \cos x.$

Recall that the steady-state solution is of the form $u_s(x) = C + Dx$ and the constants C and D can be found easily from the boundary conditions.

18.3 **6**(a,d,g). *In (d) find A_n by comparing coefficients, not by integration.*

Wave equation. 19.2 **5**(b,c,d).

Laplace equation. 20.2 **1**(a,h).

Dirichlet problem for circular disk. Consider the problem
$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ \ u(b,\theta) = f(\theta),\ (0 \le r < b,\ -\infty < \theta < \infty),$
where $f(\theta)$ is a periodic function given over one period by
$f(\theta) = 0,\ 0 < \theta < \pi/2,\ \ 10,\ \pi/2 < \theta < \pi,\ \ \text{and } 30,\ \pi < \theta < 2\pi.$

(a) Find $u(0,0)$. *You do not need to find $u(r,\theta)$ for all r and θ.*

(b) Give an upper and a lower bound for $u(r,\theta)$ in the disk, i.e. give numbers m and M such that $m \le u(r,\theta) \le M$ for all $0 \le r \le b$, $-\infty < \theta < \infty$.