

Math 508 Exam 1 – Review

Exam 1: Wednesday 9/29; covers Sections 17.2-17.8. No books, no calculators, and no notes except for one standard sheet are allowed on the test.

Review problems:

1. f is a periodic function defined over one period as follows:

$$f(x) = 0 \text{ on } [0, 2) \text{ and } 5 \text{ on } [2, 3).$$

- (a) Work out the Fourier series of f .
- (b) At which values of x does the series fail to converge to $f(x)$?
What does it converge to at those points?
- (c) Work out the complex exponential form of the Fourier series.

2. f is a function on $(0,3)$: $f(x) = 0$ on $(0, 2)$ and 5 on $(2, 3)$.

- (a) Make labeled sketches of its HRC, HRS, QRC, and QRS extensions.
- (b) Work out its HRC, HRS, QRC, and QRS expansions.

3. Derive a particular solution of the equation

(a) $x'' + x = f(t)$, where $f(t) = |\cos t|$.

(To find $\int \cos t \cos(at)$ use integration by parts twice, or use trigonometric formulas.)

(b) $x'' + x' + x = f(t)$, where $f(t)$ is a periodic function given over one period by $f(t) = 1 - t$ on $[0, 1)$ and $t - 1$ on $[1, 2)$.

Sketch f and find its Fourier series;

expand x , x' , and x'' as Fourier series;

substitute the series into the equation;

equate the corresponding coefficients to find FS x ;

calculate FS x' and FS x'' and check that they converge uniformly

(i.e. that for large n , their coefficients behave like $\frac{c}{n^k}$ where $k > 1$.)

4. Vector space approach.

Consider the space $C_p[-l, l]$ with inner product $\langle f, g \rangle = \int_{-l}^l f(x)g(x) dx$.

- (a) Give an orthogonal basis of $C_p[-l, l]$.
- (b) Give an orthonormal basis of $C_p[-l, l]$.
- (c) Let f be a function in $C_p[-l, l]$. Give the orthogonal projection of f onto the subspace spanned by $1, \cos \frac{k\pi x}{l}, \sin \frac{k\pi x}{l}, 1 \leq k \leq n$.
(Note that this projection is the best approximation of f within the subspace.)
- (d) Compute the norm of $g(x) = 1$ and $h(x) = x$. Verify that g and h are orthogonal.
Find the orthogonal projection of $f(x) = e^x$ onto the subspace spanned by g and h .

5. Sturm-Liouville theory. **Hw 3** due Wednesday, 9/29: 17.7 #1(a,c), 4(a); 17.8 #2(a).