

Math 508 Exam 2 Review

Exam 2 will be on **Monday, November 8**. It will cover sections

- 17.10 Fourier transform
- 18.2 Linear / non-linear equations, classification of second-order linear equations.
- 18.3 Diffusion equation: separation of variables.

No calculators, books, or notes, except for one standard sheet, are allowed on the test.

Review problems:

1. Use the definition of Fourier transform and inverse Fourier transform to find

- (a) $F\{H(x)e^{-ax}\}$, $Re(a) > 0$ (b) $F\{H(x+5) - H(x-5)\}$
(c) $F^{-1}\{\pi e^{-|w|}\}$

2. Use properties of Fourier transform and the table on p.1274 to find

- (a) $F\{\frac{1}{4x^2+1} + 7e^{-x^2}\}$ (b) $F\{4x^2e^{-x^2}\}$
(c) $F^{-1}\{5\omega e^{-4\omega^2}\}$ (d) $F^{-1}\{e^{-\omega^2+2\omega-1} + \frac{1}{1+2i\omega}\}$

Note: a list of Fourier transforms will be provided on the exam.

3. Is the following equation linear or non-linear? Justify your answer.

- (a) $(\cos x)u_{xx} = u_y$ (b) $u_{xx} + u u_y = 0$

4. Classify the following equations defined over $-\infty < x < \infty$ and $-\infty < y < \infty$, as elliptic, parabolic, or hyperbolic. If the equation is of mixed type, identify the relevant regions and give a classification within each region.

- (a) $u_{xx} + u_{xy} + u_{yy} + u_x + u_y + u = 1$ (b) $u_{xx} + (\cos x)u_{yy} = 2xy$

5. Find the steady-state solution of the problem

$$\alpha^2 u_{xx} = u_t, \quad u(0, t) + u_x(0, t) = 4, \quad u(2, t) = 7, \quad u(x, 0) = 1.$$

6. Solve the equation $\alpha^2 u_{xx} = u_t$ with given boundary and initial conditions.

Use separation of variables. When applying the initial condition, you may use H/Q R S/C expansions, Sturm-Liouville theory, or compare coefficients (if possible).

- (a) $u(0, t) = 2, u(3, t) = 5, u(x, 0) = 10$
(b) $u_x(0, t) = 0, u_x(2\pi, t) = 0, u(x, 0) = 0$ for $0 < x < \pi$, and 1 for $\pi < x < 2\pi$
(c) $u(0, t) = 0, u_x(3\pi, t) = 0, u(x, 0) = 3 \sin(\frac{7x}{6}) + \sin(\frac{21x}{6})$