Math 508 Final Exam Review

Final Exam will be on **Monday, December 13, 6-8 p.m.** The final will be cumulative, but the material covered after the second exam will be emphasized.

No calculators, books, or notes, except for three standard sheets, are allowed on the test.

Review problems:

1. Fourier series. A periodic function f is given over one period as follows
(a) $f(x) = |x|$ on $(-2\pi, 2\pi]$
(b) $f(x) = 2$ on $0 \le x < 3$, and 8 on $3 \le x < 6$
(c) $f(x) = 1 + 3\sin(2x)$ on $[-\pi, \pi)$

Sketch the graph of f. Is f even, odd, or neither even nor odd?

Work out the Fourier series of f. At which values of x, if any, does the series fail to converge to $f(x)$? To what values does it converge at those points?

2. Fourier transform. *A list of Fourier transforms will be provided on the exam.* Use properties of the Fourier transform and the table on p.1274 to find

(a) $F\{xe^{-x^2} - H(x+5) + H(x-5)\}$ (b) $F^{-1}\{e^{-(w+3)^2} + \frac{5}{1-3i\omega}\}$

3. Diffusion equation. Find the steady-state solution of the problem
$\alpha^2 u_{xx} = u_t,\ 2u(0,t) - u_x(0,t) = 5,\ u(4,t) + u_x(4,t) = 3,\ u(x,0) = \sin x.$

4. Wave equation. Solve for $y(x,t)$ by separation of variables.

(a) $c^2 y_{xx} = y_{tt},\ y(0,t) = 0,\ y(\pi,t) = 0,\ y(x,0) = 0,\ y_t(x,0) = 2.$
(b) $c^2 y_{xx} = y_{tt},\ y_x(0,t) = 0,\ y(2,t) = 0,\ y(x,0) = f(x),\ y_t(x,0) = 0.$

Leave the coefficients in (b) in integral form.

5. Show that the equation $u_{xx} + 6u_{xy} + 8u_{yy} = 0$ is hyperbolic and find its general solution by making the change of variables $\xi = 4x - y$, $\eta = 2x - y$.

6. Laplace equation. 20.2 Ex. 1(d,n)

7. Dirichlet problem for circular disk and harmonic functions.
Consider the problem

$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ u(b,\theta) = f(\theta),\ (0 \le r < b,\ -\infty < \theta < \infty),$
where $f(\theta)$ is a periodic function given over one period by
$f(\theta) = 20,\ 0 < \theta < \pi$, and $f(\theta) = 40,\ \pi < \theta < 2\pi$.

(a) Find $u(0,0)$. (You do not need to solve the problem completely.)
(b) Is it possible that $u(r,\theta) = 10$ for some r, $0 \le r < b$, and some θ?