

## MA 535      STUDY GUIDE FOR THE FINAL EXAM

The exam will be cumulative but the following topics will be emphasized:

- Series;
- Continuity;
- Differentiation;
- Riemann-Stieltjes integral.

You will be asked to give definitions, state theorems. give proofs of some of the theorems, and solve problems. The latter may include T/F questions where you need to give a proof or a counterexample.

Here is a list of definitions and theorems that you need to know.

### Definitions.

- Real numbers (axiomatic definition)
- Metric space
- Neighborhood, limit point, interior point
- Open and closed sets
- Bounded set
- Separated sets, connected set
- Compact set
- Convergent sequence in a metric space
- Cauchy sequence in a metric space
- $\limsup$  and  $\liminf$  of a sequence of real numbers
  
- Convergence of a series
- The number  $e$  (two definitions)
- Power series
- Absolute convergence
- Limit of as function
- Continuity of a function
- Uniform continuity
- Differentiability and derivative
- Riemann integral
- Riemann-Stieltjes integral

## Theorems.

You need to know the statements of the theorems below.

You should also be able to prove the results marked by \*.

- Theorems about open, closed, and connected sets:  
2.23\*, 2.24, 2.47 and a description of connected sets in  $\mathbb{R}$ .
  
- Theorems about compact sets:
  - Any compact set in a metric space is closed and bounded.
  - Closed subsets of compact sets are compact \*.
  - A set in  $\mathbb{R}^k$  is compact  $\Leftrightarrow$  it is closed and bounded.
  - A set  $K$  in a metric space is compact  $\Leftrightarrow$   
any infinite subset of  $K$  has a limit point in  $K$ .
  - Theorem 2.42 (Weierstrass).
  
- Theorems about sequences: 3.2\*, 3.3\*, 3.6, 3.11, 3.14, 3.20.
  
- Theorems about series: 3.22, 3.23, 3.24, 3.25\*, 3.26, 3.28, 3.33\*, 3.34\*, 3.39, 3.43, 3.45, 3.47, 3.50, 3.54, 3.55\*.
  
- Theorems about limits and continuity: 4.2\*, 4.4, 4.6, 4.7\*, 4.8\* and Corollary, 4.9, 4.10, 4.14\*, 4.15, 4.16, 4.17\*, 4.19, 4.20, 4.22\*, 4.23, 4.29, 4.30\*.
  
- Theorems about differentiation: 5.2\*, 5.3\*, 5.5, Derivative of the inverse function, 5.8, Rolle's Theorem, 5.10, 5.11.
  
- Theorems about integration: 6.4, 6.5, 6.6\*, 6.8\*, 6.9, 6.10, 6.12\*, 6.13, 6.20\*, 6.21\*.