MA 535 Homework 12

due Wednesday, 11/28

Chapter 5 Ex. **1, 3, 5, 6**.

A. Let f be a differentiable function on (a, b). Is it true that

(i) If $f(x) \to 0$ as $x \to \infty$ then $f'(x) \to 0$ as $x \to \infty$.

(ii) If $f'(x) \to 0$ as $x \to \infty$ then there is a number c such that $f(x) \to c$ as $x \to \infty$.

B. Let f be a differentiable function on (a, b). Is it true that

(i) If f is strictly monotonically increasing on (a, b) then $f'(x) > 0$ for all x in (a, b).

(ii) If $f'(x) > 0$ for all x in (a, b) then f is strictly monotonically increasing on (a, b).

C. Let f and g be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$.
Suppose that $f(0) = g(0)$ and $f'(x) \geq g'(x)$ for all $x \in (0, \infty)$.
Prove that $f(x) \geq g(x)$ for all $x \in [0, \infty)$.

D. Let $f(x) = x^4 + x^2 - 2$.

(i) Prove that f is strictly monotonically increasing on $[0, \infty)$.

(ii) Find $f([0, \infty))$.

(iii) Find $(f^{-1})'(0)$ and $(f^{-1})'(18)$.