

MA 535 Homework 9

due Wednesday, 10/31

Chapter 3 Ex. 9.

1. Is it true that

- (a) If $c_n \rightarrow \infty$, then the radius of convergence of $\sum c_n z^n$ is at most 1?
- (b) If $c_n \rightarrow \infty$, then the radius of convergence of $\sum c_n z^n$ is 0?

2. Is it true that

- (a) If $\sum(a_n + b_n)$ converges, then $\sum a_n$ and $\sum b_n$ converge?
- (b) If $\sum(a_n + b_n)$ and $\sum a_n$ converge, then $\sum b_n$ converges?

3. Consider a series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ such that $a_n \geq 0$ for all n , $a_n \geq a_{n+1}$ for all n , and $a_n \rightarrow 0$. Prove that for any $N \geq 1$, $|\sum_{n=N}^{\infty} (-1)^{n-1} a_n| \leq a_N$, i.e. the “tail” can be estimated by its first term.

4. Give an example of a divergent series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ such that $a_n \geq 0$ for all n and $a_n \rightarrow 0$ (but $\{a_n\}$ is not necessarily monotone decreasing).

5. Is it true that if a series $\sum a_n$ converges and a sequence $\{b_n\}$ converges to 0, then the series $\sum a_n b_n$ converges?