MA 535 STUDY GUIDE FOR MIDTERM EXAM

Midterm Exam will be on **Wednesday, October 17.**
No books or notes will be allowed on the test.

The exam will cover the following topics:

- Real numbers.
- Countable and uncountable sets.
- Basic topology.
- Sequences.

You will be asked to

- Give definitions.
- State theorems.
- Give proofs of some of the theorems.
- Solve problems. This may include T/F questions where you need to give a proof or a counterexample. Make sure that you know how to solve all Hw problems.

Here is a list of definitions and theorems that you need to know.

Definitions.

- Field
- Ordered set, ordered field
- The least upper bound property
- Real numbers (axiomatic definition)
- Countable, at most countable, uncountable sets
- Cantor set
- Metric space
- Convex set in $\mathbb{R}^k$
- Neighborhood, limit point, interior point
- Open and closed sets
- Bounded set
- Dense set
- Closure, interior, boundary
- Separated sets, connected set
- Compact set
- Convergent sequence in a metric space
- Cauchy sequence in a metric space
- Complete metric space
- $\limsup$ and $\liminf$ of a sequence of real numbers

Theorems. You need to know the statements of the theorems below.
You should also be able to prove the results marked by *.

- Theorems about countable sets, etc.
  - 2.8, 2.12 and its corollary, 2.14*.
  - If A and B are countable then $A \times B$ is countable.
  - $\mathbb{Q}$ is countable, Cantor set is uncountable.
  - A non-empty set X and its power set (the set of all subsets) are not equivalent.
- Theorems about limit points, open and closed sets, etc.
  - Theorems 2.19*, 2.20 and corollary, 2.22, 2.23*, 2.24*, 2.30.
  - Theorem 2.27 and its counterpart for the interior (p.43 Ex. 9 (a-c)).
- Connected sets: Theorem 2.47 and a description of connected sets in $\mathbb{R}$.
- Theorems about compact sets:
  - Any compact set in a metric space is closed and bounded *.
  - Theorems 2.38 and 2.39 about nested closed intervals and nested k-cells.
  - Theorem 2.40*: any k-cell is compact.
  - Closed subsets of compact sets are compact *.
  - A set in $\mathbb{R}^k$ is compact $\Leftrightarrow$ it is closed and bounded.
  - A set K in a metric space is compact $\Leftrightarrow$ any infinite subset of K has a limit point in K. (We proved $\Rightarrow$ for any metric space and $\Leftarrow$ only for $\mathbb{R}^k$)
  - Theorem (Weierstrass): Every bounded infinite subset of $\mathbb{R}^k$ has a limit point in $\mathbb{R}^k$.
  - If $\{K_n\}$ is a sequence of non-empty compact sets such that $K_1 \supset K_2 \supset \dots$ then $\cap_{n_1}^{\infty} K_n$ is non-empty.
  - Let $K \subset Y \subset X$. Then K is compact relative to $Y \Leftrightarrow K$ is compact relative to X.
- Theorems about convergent sequences:
  - 3.2*, 3.3*, 3.4*, and 3.20.
  - 3.14*: Any monotone bounded sequence in $\mathbb{R}$ converges.
- Theorem 3.6 about subsequences.
- Theorem 3.11 about Cauchy sequences.