MA 536 STUDY GUIDE FOR THE FINAL EXAM

Final Exam will be on **Wednesday, April 30, 10:30-12:30, in ILB 345.**

No books or notes will be allowed on the test.

The exam will be cumulative: it will cover Chapters 7, 8, 9, and 11, but the material of Chapter 11 will be emphasized. You will be asked to give definitions, state theorems, give proofs of some of the theorems, and solve problems. This may include T/F questions where you need to give a proof or a counterexample. Make sure that you know how to solve all Hw problems.

Here is a list of definitions and theorems that you need to know.

Definitions.

- Pointwise convergence of a sequence of functions
- Uniform convergence of a sequence of functions
- Pointwise convergence of a series of functions
- Uniform convergence of a series of functions
- The supremum norm
- Equicontinuous family of functions

- Power series
- Taylor polynomials and Taylor series
- e^x as a power series
- Fourier series
- Orthogonal / orthonormal system
- L^2-norm

- Linear transformation
- Norm of a linear transformation
- Differentiable function $\mathbb{R}^n \to \mathbb{R}^m$ and its derivative
- Partial derivatives and gradient
- Directional derivatives
- Continuously differentiable function
- Contraction

- Ring, σ-ring
- Additive, countably additive set function.
- Elementary sets
- Regular set function
- Outer measure
- Finitely μ-measurable sets and μ-measurable sets.
- Lebesgue measure
- Borel sets
- Measurable space
- Measurable function
- Simple function
- Lebesgue integral

Theorems. You need to know the statements of the theorems below.
You should also be able to prove the results marked by *.

- Chapter 7: Sequences and series of functions.

  7.8, 7.10, 7.12*, 7.16, 7.17; three theorems for series of functions: continuity of the sum, integration, and differentiation; 7.24*, 7.25, 7.26.

- Chapter 8: Some special functions.

  8.1* and Corollary, 8.11*, 8.12*;

  If $\{\phi_n\}$ is a complete ortnonormal system in $L^2[a,b]$ then for any $f \in L^2[a,b]$ the Fourier series of f relative to $\{\phi_n\}$ converges to f in the L^2-norm.

  If f is a continuously differentiable 2π-periodic function then its Fourier series converges to f uniformly.

- Chapter 9: Functions of several variables.

  9.7*, 9.8, 9.12*, 9.15, 9.17, 9.19, 9.21, 9.23*, 9.24, 9.27, 9.28.

- Chapter 11: The Lebesgue theory

  11.8*, 11.10,

  Borel sets are μ-measurable for every μ,

  Any μ-measurable set is a union of a Borel set and a set of measure zero,

  11.15*, 11.16, 11.17* and Corollaries, 11.18, 11.20*,

  11.23, 11.24, 11.26*, 11.27, 11.28*, 11.29, 11.30, 11.32 and Corollary, 11.33.