

MA 536 Homework 1

due Wednesday, 1/16

Chapter 7 Ex. 1, 2.

A. Let $f_n(x) = \cos\left(x + \frac{1}{n}\right)$. Prove that $\{f_n\}$ converges to $f(x) = \cos x$ uniformly on \mathbb{R} .

Hint: Use the Mean Value Theorem.

B. Is it possible for a sequence of discontinuous functions to converge to a continuous function uniformly on $[a, b]$? Justify your answer.

C. Let $f_n(x) = \frac{x}{n}e^{-x/n}$.

(i) Prove that $\{f_n\}$ converges to 0 pointwise on $[0, \infty)$.

(ii) Prove that the convergence is not uniform on $[0, \infty)$.

D. Let $f_n(x) = e^{-(x-n)^2}$ on $(-\infty, \infty)$. Sketch the graph of f_n and determine from the picture whether the sequence converges pointwise, and if it does whether the convergence is uniform on $(-\infty, \infty)$. No explanations are required.