# MA 536 Homework 9

due Wednesday, 4/2

**1.** For each of the following collections of sets determine whether it is a ring. If it is just answer *Yes*, if it is not explain why.

(a) $\mathcal{R} = \{$ finite unions of semi-open intervals $[\,a, b\,)\,\}$;

(b) $\mathcal{R} = \{$ finite unions of closed intervals $[\,a, b\,]\,\}$.

**2.** For each of the following collections of sets determine whether it is a $\sigma$-ring. If it is just answer *Yes*, if it is not explain why.

(a) $\mathcal{R} =$ the collection of all at most countable sets in $\mathbb{R}$;

(b) $\mathcal{R} =$ the collection of all cocountable sets in $\mathbb{R}$ together with the empty set (a set is called cocountable if its complement is at most countable);

(c) $\mathcal{R} =$ the union of the collections in (a) and in (b).

**3.** Let $f$ be an additive non-negative set function on a ring $\mathcal{R}$. Prove that for any $A$,$B \in \mathcal{R}$

(a) $f(A \cup B) + f(A \cap B) = f(A) + f(B)$;

(b) If $B \subset A$ then $f(B) \leq f(A)$;

(c) If $B \subset A$ and $f(B) \neq \infty$ then $f(A - B) = f(A) - f(B)$.

**4.** Let $\mathcal{E}$ be the collection of elementary sets in $\mathbb{R}$, and let $m$ be the length on $\mathcal{E}$. Prove that $m$ is regular on $\mathcal{E}$.

**5.** Let $C$ be the standard Cantor set, and let $m^*$ be the outer measure corresponding to the length $m$. Using the definition of $m^*$, prove that $m^*(C) = 0$.