

MATH IN ACTION Student Seminar

PROBLEM SET 1

Can you solve these problems?

Bring your ideas to the seminar on Tuesday, October 3, at 11 a.m. in ILB 430.

We will work on the problems together! Refreshments will be served.

1. There are 30 students in a Math class, and some of them are friends. Prove that at least two of the students have the same number of friends in this class.
2. 43 identical balls move along a line, 26 of them from left to right with speed v , and the remaining 17 from right to left toward the first group of balls with speed w . When two balls collide, they exchange their speeds and directions of motion. What is the total number of collisions that will occur?
3. Three roommates, Jack, Jim, and John, could not decide who will clean the house after a party. So they put 123 matches on the table and agreed that they will draw in turn 1, 2, or 3 matches from the pile, and the one who takes the last match will do the cleaning. Jack gets the first turn, Jim draws second, and John takes last. John does not trust Jim on this matter and never draws the same number of matches as Jim has drawn immediately before him. Who can avoid cleaning the house regardless of how the others draw the matches? How can he do this?
4. A storage room is a cube whose side is 8 feet. A spider sees a fly in the center of the ceiling from a lower corner of the room. What is the shortest distance the spider needs to travel to get to the fly?
5. Prove that $|\cos a - \cos b| \leq |a - b|$ for all a and b .
6. Consider the region bounded by the parabola $y = x - x^2$ and the x -axis. There is a line through the origin that divides this region into two regions with equal area. What is the slope of this line?
7. The surface of a container is obtained by rotating the graph of a function $f(x)$ about the y -axis. There is a small hole in the bottom of the container through which water can flow. It is known that the water flows through the hole at a rate proportional to the square root of the height of the water level in the container. Find the function $f(x)$ so that the water level in the container would decrease at a constant rate.
8. Find the limit

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} + \sin \frac{n\pi}{n} \right)$$

9. For which values of p does the series converge?

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^p + (-1)^n}$$