

1. Do Problems 2 on page 229.
2. Do Problem 3 on page 229.
3. Do Problem 5 on page 229.
4. Do Problem 8 on page 229.
5. Recall from Problem 8 on page 193 that  $G = \text{GL}_n\mathbb{R}$  operates on  $\mathbb{R}^n$  by left multiplication, and the stabilizer  $G_{e_1}$  is the subgroup of  $G$  consisting of all matrices of the form

$$\begin{pmatrix} 1 & v_0^t \\ 0 & M_0 \end{pmatrix},$$

where  $v_0$  is an arbitrary vector in  $\mathbb{R}^{n-1}$ ,  $v_0^t$  is its transpose (so that we can think of it as a row vector rather than a column vector),  $0$  is the zero vector in  $\mathbb{R}^{n-1}$  and  $M_0 \in \text{GL}_{n-1}\mathbb{R}$ .

As a set,  $G_{e_1}$  is in obvious one-to-one correspondence with  $\mathbb{R}^{n-1} \times \text{GL}_{n-1}\mathbb{R}$ . Find a multiplication rule that will make  $\mathbb{R}^{n-1} \times \text{GL}_{n-1}\mathbb{R}$  a group that is isomorphic to  $G_{e_1}$ . Verify the group axioms. Hint: Consider the product of two arbitrary matrices of  $G_{e_1}$  to determine the multiplication. Use block matrix multiplication or else let  $n$  be a small, specific number.