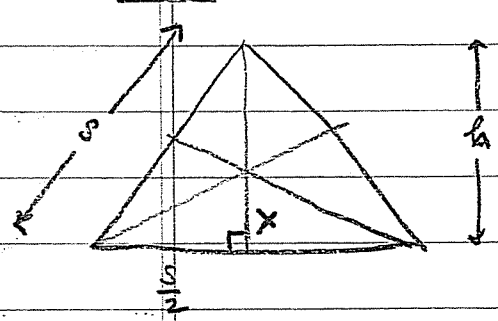
MA126 Solution to #17, p.390

Problem: Find the volume of a regular tetrahedron with edge length $s$.

Solution Each face is an equilateral triangle.




$\frac{x}{s/2} = \tan\frac{\pi}{6}$, so $x = s/2\sqrt{3}$

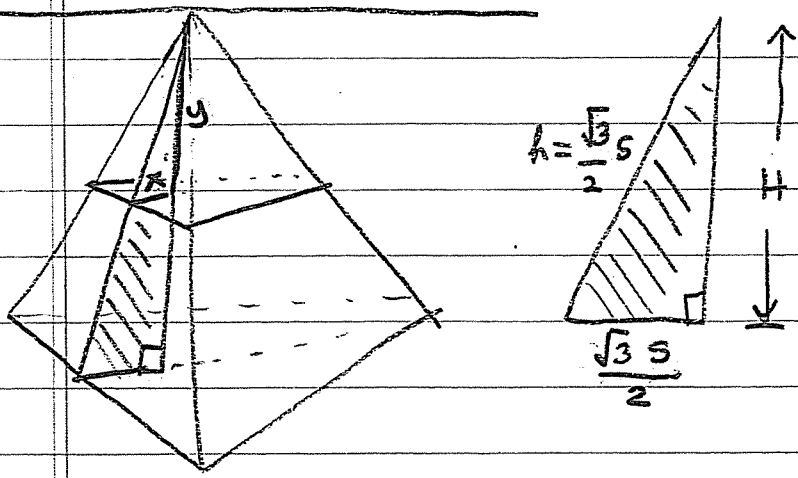
By the Pythagorean Theorem,

$\left(\frac{s}{2}\right)^2 + h^2 = s^2$, so $h = \frac{\sqrt{3}}{2}s$

We can express the area in terms of $x$:

$$A(x) = \frac{1}{2}sh = \frac{\sqrt{3}}{4}s^2 = 3\sqrt{3}x^2$$

Now consider the solid:




$H = \sqrt{\frac{2}{3}}s$ by the Pythagorean Theorem

By similar triangles, $\frac{x}{s/2\sqrt{3}} = \frac{y}{s\sqrt{2}/\sqrt{3}}$, so $x = \frac{y}{2\sqrt{2}}$.

$$\text{Volume} = \int_0^H A(x)\,dy = \int_0^{s\sqrt{\frac{2}{3}}} 3\sqrt{3}\left(\frac{y}{2\sqrt{2}}\right)^2 dy = \cdots = \frac{\sqrt{2}}{12}s^3$$