MA 126 SOLUTIONS TO TEST 2

① a) $\int \underset{u}{x}\, \underset{v'}{\sec^2 x}\, dx = x\tan x - \int \tan x\, dx = x\tan x + \int \frac{-\sin x}{\cos x}\, dx$

$u' = 1 \quad v = \tan x$

$= x\tan x + \ln|\cos x| + C$

b) $\int \underset{u}{\tan^{-1} x}\, dx = x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$

$v' = 1$

$u' = \frac{1}{x^2+1} \quad v = x$

$= x\tan^{-1}x - \frac{1}{2}\ln(x^2+1) + C$

② a) $\int \cos^2\theta\, d\theta = \int \frac{1}{2}(1+\cos 2\theta)\, d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C$

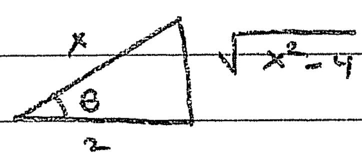
b) $\int_0^{\pi/6} (\sin\theta)^3 \cos\theta\, d\theta = \left.\frac{(\sin\theta)^4}{4}\right|_0^{\pi/6} = \frac{1}{4}\left(\frac{1}{2}\right)^4 = \frac{1}{64}$

③ $\int \frac{x^2\, dx}{\sqrt{4-x^2}} = \int \frac{4\sin^2\theta\ \cancel{2\cos\theta}\, d\theta}{\cancel{2\cos\theta}} = 4\int \sin^2\theta\, d\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

④ $\sec\theta = \frac{x}{2}$



; $\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{16}\sec^{-1}\frac{x}{2} + \frac{1}{32}\cdot 2\,\frac{\sqrt{x^2-4}}{x}\,\frac{2}{x} + C$

$= \frac{1}{16}\sec^{-1}\frac{x}{2} + \frac{1}{8}\,\frac{\sqrt{x^2-4}}{x^2} + C$

⑤ $\frac{1}{(x-3)(x-7)} = \frac{A}{x-3} + \frac{B}{x-7}$

$1 = A(x-7) + B(x-3)$

let $x=3$: $1 = -4A$; $A = -1/4$

$x = 7$: $1 = 4B$; $B = 1/4$

$\frac{-1}{4(x-3)} + \frac{1}{4(x-7)}$

(6) a) $\lim_{B\to\infty} -\frac{1}{2}\int_0^B -2x\, e^{-x^2}\,dx = -\frac{1}{2}\lim_{B\to\infty} e^{-x^2}\Big|_0^B$

$= \frac{1}{2}\lim_{B\to\infty} e^{-x^2}\Big|_B^0 = \frac{1}{2}\lim_{B\to\infty}\left(1 - e^{-B^2}\right) = \frac{1}{2}$, where $e^{-B^2} \to 0$

b) $\lim_{B\to 1^+}\int_B^2 \frac{dx}{(x-1)^2} = \lim_{B\to 1^+}\int_B^2 (x-1)^{-2}\,dx$

$= \lim_{B\to 1^+} \frac{(x-1)^{-1}}{-1}\Big|_B^2 = \lim_{B\to 1^+} (x-1)^{-1}\Big|_2^B$

$= \lim_{B\to 1^+} (B-1)^{-1} - 1$, where $(B-1)^{-1} \to \infty$ does not exist

(7) a) $f(x) = e^{-x}$ $f'(x) = -e^{-x}$ $f''(x) = e^{-x}$ etc

$f(0) = 1$ $f'(0) = -1$ $f''(0) = 1$ etc

$T_4(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$

b) This problem should have read: Find the maximum possible size of $|e^{-0.1} - T_4(0.1)|$

$|f^{(5)}(x)| = e^{-x}$ Consider the interval $[0, 0.1]$

Error $\leq \frac{K\,(0.1)^5}{5!} = \frac{1\cdot(0.1)^5}{5!}$

Full credit (5 points) was given for any answer that demonstrated a knowledge of the correct error formula.

(8) a) $a_n = \sqrt{\frac{9 + 5/n}{1 + 3/n^2}} \longrightarrow \sqrt{9} = 3$, where $5/n \to 0$ and $3/n^2 \to 0$

b) $b_n = \frac{\sin\frac{1}{n}}{\frac{1}{n}} \longrightarrow \lim_{x\to 0}\frac{\sin x}{x} = 1$

(9) a) $S_1 = \frac{1}{\sqrt{1+1}} - \frac{1}{\sqrt{2+1}}$

$S_2 = \left(\frac{1}{\sqrt{1+1}} - \cancel{\frac{1}{\sqrt{2+1}}}\right) + \left(\cancel{\frac{1}{\sqrt{2+1}}} - \frac{1}{\sqrt{3+1}}\right)$

$S_N = \frac{1}{\sqrt{1+1}} - \frac{1}{\sqrt{N+2}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{N+2}}$

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+2}} = \lim_{N\to\infty} S_N = \frac{1}{\sqrt{2}}$

(10) a) $5\left(1 - \frac{1}{4} + \frac{1}{4^2} - \frac{1}{4^3} + \cdots\right) = 5\sum_{n=0}^{\infty}\left(-\frac{1}{4}\right)^n$

$= \frac{5}{1-(-\frac{1}{4})} = \frac{5}{5/4} = 4.$

b) $\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n = \frac{1}{1-\frac{2}{5}} = \frac{5}{5-2} = \frac{5}{3}$

(11) a) $\sum_{n=0}^{\infty} \frac{3n+1}{n+3}$ $\frac{3n+1}{n+3} \to 3 \neq 0$ The series diverges since it's general term $\not\to 0$.

b) $\frac{1}{n^5+n} \leq \frac{1}{n^5}$. Since $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges

(p-series, $p = 5 > 1$) $\sum_{n=1}^{\infty} \frac{1}{n^5+n}$ converges by CT.

c) $\frac{1}{\ln n} \geq \frac{1}{n}$ for $n \geq 2$.

Since $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges (it's the harmonic series)

$\sum_{n=2}^{\infty} \frac{1}{\ln n}$ diverges by CT