MA 126 QUIZ 11 April 22, 2010

Name *Solutions*

"There is no remainder in the arithmetic of infinity. There is only the blanket." Show your work.

1. Find the radius R of convergence of the power series. Justify your answer.

[5] a) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n x^n}{n^2}$ a_n

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5^{n+1} x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{5^n x^n}\right|$$

$$= 5 \frac{n^2}{(n+1)^2} |x| \longrightarrow 5|x| \overset{\text{Want}}{<} 1$$

$$|x| < \frac{1}{5} \qquad R = \frac{1}{5}$$

[5] b) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+5)^n}{(n+1)!}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x+5)^{n+1}}{(n+2)!} \frac{(n+1)!}{(x+5)^n}\right|$$

$$= \frac{1}{n+2} |x+5| \longrightarrow 0 < 1 \quad \text{for all } x$$

$$R = \infty$$

2. The radius of convergence of the power series is given. Find the interval of convergence. Justify your answer.

[5] a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{n^2+1}$, $R = 1$.

? ?

0 1 2

Let $x = 0$ $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges by IT or comparison with p-series $\sum \frac{1}{n^2}$

$x = 2$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ converges by AST $[0, 2]$

[5] b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n\,2^n}$, $R = 2$.

-2 0 2

Let $x = -2$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n\,2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by AST

$x = 2$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n\,2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges $[-2, 2)$