MA 126 QUIZ 9 March 25, 2010

Name Solutions

A lazy spirit is a losing spirit. Show your work.

1. Determine whether the improper integral converges. If so, find it. If not, justify your answer.

[5] a) $\int_2^\infty e^{-2x}\,dx = \lim_{B\to\infty} \int_2^B e^{-2x}\,dx = \lim_{B\to\infty} \left(-\frac{1}{2}e^{-2x}\Big|_2^B \right)$

$= \lim_{B\to\infty} \left(\frac{1}{2} e^{-2x}\Big|_B^2 \right) = \lim_{B\to\infty} \left(\frac{1}{2}e^{-4} - \frac{1}{2}\underbrace{e^{-2B}}_{\to 0} \right)$

$= \frac{1}{2}e^{-4}$

[5] b) $\int_1^2 \frac{dx}{x\ln x} = \lim_{B\to 1^+} \int_B^2 \frac{1}{\ln x}\frac{1}{x}\,dx$

$= \lim_{B\to 1^+} \left(\ln(\ln x)\Big|_B^2 \right) = \lim_{B\to 1^+} \left(\ln(\ln 2) - \ln(\underbrace{\ln B}_{\to 0}) \right)$

does not exist

2. [5] Let $f(x) = 1/x$. Find an expression for $f^{(n)}(1)$.

$f(x) = x^{-1}$; $f(1) = 1$

$f'(x) = -x^{-2}$; $f'(1) = -1$

$f''(x) = 2x^{-3}$; $f''(1) = 2$

$f'''(x) = -6x^{-4}$; $f'''(1) = -6$

$f^{(n)}(1) = (-1)^n n!$

3. [5] Find the Maclaurin polynomial $T_4(x)$ for $f(x) = \cos x$.

$f(x) = \cos x$; $f(0) = 1$

$f'(x) = -\sin x$; $f'(0) = 0$

$f''(x) = -\cos x$; $f''(0) = -1$

$f'''(x) = \sin x$; $f'''(0) = 0$

$f^{(4)}(x) = \cos x$; $f^{(4)}(0) = 1$

$T_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} = 1 - \frac{x^2}{2} + \frac{x^4}{24}$

Bonus [5] Find the maximum possible size of $|\cos(0.1) - T_4(0.1)|$. Leave your answer in a form ready for a calculator.

$|\cos(0.1) - T_4(0.1)| \le \frac{1\,(0.1)^5}{5!}$ since $|f^{(5)}(x)| = |-\sin x| \le 1$ for all x