Unit 16

- We can prove conclusions & theorems in PL but how can we show an argument is invalid?
- The counterexample method
- How does this work for PL?
- Natural interpretation method
  - Assign meanings to propositional functions and individual constants such that the premises are true and the conclusion is false.
- The model universe method
  - Assign *formal* meanings to propositional functions and individual constants such that premises are true and conclusion false.

Objectives

- Be able to demonstrate the invalidity of quantifier arguments using the natural interpretation method.
- Learn the definition of truth for the universal and existential quantifiers
- Be able to demonstrate the invalidity of quantifier arguments using the model universe method.

Natural Interpretation Method

- Intuitive counterexample
  - Every famous logic professor has bad hand writing. Frege had bad hand writing. So, Frege is a famous logic professor. (T, T, T)
  - (x)(Lx⊃Bx), Bf /:. Lf
  - Every famous logic professor has bad hand writing. Poston has bad hand writing. So, Poston is a famous logic professor. (T, T, F)
- Procedure for Natural Interpretation Method
  - Specify the domain of discourse,
    - i.e., the set of objects over which the bound variables range
    - i.e., the things to which the formulas could be refering
    - ➢ PL is a "first-order" logic because the bound variables range only over *objects (or individuals)*.
    - ➢ In a "second-order" logic the bound variables range over individuals *and* properties

Natural Interpretation Method

- Procedure for Natural Interpretation Method
  - Specify the domain of discourse
  - Reinterpret the predicate letters by assigning to them propositional functions that make the premises true and the conclusion false.
- Example:
  - All communists are in favor of socialized medicane and all socialists are in favor of socialized medicine, so all socialists are communists.
  - $(x)(Cx \supset Fx)$, $(x)(Sx \supset Fx)$ /∴ $(x)(Sx \supset Cx)$
  - Domain = {human beings}
  - Cx: x is a normal man
  - Sx: x is a normal woman
  - Fx: x has a brain

Natural Interpretation Method

- Why the domain matters
- Sentences can be true in one domain but false in another
- Wx: x walks before the age of six months
- Px: x is precocious
- $(x)(Wx \supset Px)$
- Domain = {human beings}
  - '$(x)(Wx \supset Px)$' is true
- Domain = {mammals}
  - '$(x)(Wx \supset Px)$' is false

- To find a CE it's usually easier to restrict the domain
- Make sure the propositional function makes sense given the domain, i.e., x is red doesn't make sense in the domain of numbers
- Make sure the CE is a substitution instance of the bad form
- Make sure that there is *no question* about the truth of the premises and conclusion.
- Make explicit the domain, the meaning of the prop functions, and the counterexample.

Natural Interpretation Method

- Demonstrate the invalidity of the following
  - $(x)(Ax \supset Bx), (\exists x)(Ax \cdot Cx)$ /∴ $(x)(Cx \supset Bx)$

Model Answer

1. The domain is the entire universe
2. Ax: x is a cat; Bx: x is a mammal; Cx: x has four legs
3. (1) All cats are mammals. (2) Some cats have four legs. (3) All four legged things are mammals.

Natural Interpretation Method

- As a rule of thumb first think of a way the conclusion could be false.
- The Nat. Int. method takes imagination and creativity.
- Some cases will be easy and some very challenging.
- The model universe method is much easier.
- Practice #2 all starred

Truth Conditions for Quantifier Statements

- Model universe method requires knowledge of the truth conditions or semantics of quantifier statements
- '(x)Px' is true iff each individual in the domain has property P.
- Each instance Pa, Pb, Pc, … is true.
- '(∃x)Px' is true iff at least one individual in the domain has property P.
- One instance of Pa, Pb, Pc, … is true
- '(∃x)Px' is false just in case each instance Pa, Pb, Pc, … is false.
- Analogy between
  - universal quantifier and conjunction
  - Existential quantifier and disjunction

The Model Universe Method

- An argument is valid iff its form is valid.
- An argument's form is valid iff there's no possible substitution instance of it with true premises and a false conclusion.
- All we need to show is that there's some domain—it may not be *intuitive*—in which the premises are true and the conclusion is false.
- The model universe method makes use of this fact
- It uses highly restricted, artificial domains

The Model Universe Method

- To specify the domain all that matters is *how many individuals there are in the domain*.
- Domain of 1: {a}
- Domain of 2: {a,b}
- Domain of 3: {a, b, c}
- Given the domain, apply the definition of truth for quantifier statements within that domain.
- Rewrite universal statements as big conjunctions
- Rewrite existential statements as big disjunctions

The Model Universe Method

- Domain: {a,b}
- (x)Fx
  - Fa·Fb
- (x)((Fx∨Gx)⊃Hx)
  - ((Fa∨Ga)⊃Ha) · ((Fb∨Gb)⊃Hb)
- (∃x)Fx
  - Fa∨Fb
- (∃x)(Fx·Gx)
  - (Fa·Ga)∨(Fb·Gb)

The Model Universe Method

- Argument: Some quarks are massless and some neutrinos are massless, so some quarks are neutrinos.
- Form: $(\exists x)(Qx \cdot Mx)$, $(\exists x)(Nx \cdot Mx)$ /∴ $(\exists x)(Qx \cdot Nx)$
- Domain: {a,b}
- Argument reformulated
  - $(Qa \cdot Ma) \vee (Qb \cdot Mb)$
  - $(Na \cdot Ma) \vee (Nb \cdot Mb)$
  - /∴ $(Qa \cdot Na) \vee (Qb \cdot Nb)$
- Counterexample (via short arg method): Qa=Ma=Nb=Mb=true; otherwise false.

The Model Universe Method

- (x)(Cx⊃Bx), (x)(Cx⊃Ax), /:. (x)(Bx⊃Ax)
- Domain: {a}
- Ca⊃Ba, Ca⊃Aa, /:. Ba⊃Aa
- Ca=Aa=false, Ba=true

1. Pick a domain
2. Rewrite the quantifier statements
3. Interpret the predicate letters by assigning truth values to each simple sentence
4. Indicate how the truth table computations yield a counterexample

- See example p. 307.

The Model Universe Method

- What about TF compounds with quantifiers?
- (x)Fx⊃(x)Gx
- Domain: {a, b}
- (Fa·Fb)⊃(Ga·Gb)
- (x)Fx⊃(x)Gx
- / :. (x)(Fx⊃Gx)
- Domain: {a, b}
- (Fa·Fb)⊃(Ga·Gb)
- / :. (Fa⊃Ga)·(Fb⊃Gb)
- Counterexample: Fa=true, Ga=false, Fb=false.

The Model Universe Method

- (∃x)Fx /:. Fa
- Domain: {a,b}
- Fa∨Fb /:. Fa
- Fa=F, Fb=t
    - See p. 309
- How do you pick the size of the domain?
- If invalid in SL a domain of 1 will suffice.
- If it's not an obviously bad form in SL, pick a domain of 2, then 3, then 4,…, to the upper limit
- Where *n* is the number of different predicate letters in an argument form, the largest domain you need to test is one with 2^n individuals.
- If there is a counterexample to an argument form with *m* individuals there will be counterexamples in larger domains.
- But if there's no counterexample in a domain with 2^n individuals then there isn't a counterexample in smaller domains.