There are 7 questions. For full credit, show all steps of your work. A calculator may not be used.

1. The graph of a function $f(x)$ is shown in the graph below. Write down the values of the following limits or state that they do not exist.

   \[
   \lim_{x \to 1} f(x) = \quad \lim_{x \to 2} f(x) = \]

   \[
   \lim_{x \to 3} f(x) = \quad \lim_{x \to 5} f(x) = \]

2. Using any of the results and methods you have seen in the course, calculate the following limits (if they exist). Justify your answers.

   a. \[\lim_{x \to 1} \frac{2(x-1)^3}{(x-1)^2}\]

   b. \[\lim_{x \to 1} \frac{2(x-1)^2}{(x-1)^3}\]

3. Using any of the results and methods you have seen in the course, calculate the following limit (if it exists). Justify your answer.

   \[\lim_{s \to 4} \frac{s - 4}{\sqrt{s - 2}}\]
4. Using any of the results and methods you have seen in the course, calculate the following limits (if they exist). Show intermediate steps and justify your answers.

   a. \( \lim_{\theta \to 0} \frac{\sin(3\theta)}{5\theta} \)

   b. \( \lim_{\theta \to 0} \frac{\sin(\theta)}{\sin(2\theta)} \)

5. To prove the following limit using the formal definition of the limit, what would you need to show? (That is, write down the formal definition of the following limit in terms of “epsilon” and “delta.”) You do not need to prove the limit.

   \[ \lim_{x \to 3} (4x + 10) = 22 \]

6. Prove the limit statement given in problem #5.

   \[ \lim_{\theta \to 3} (4\theta + 10) = 22 \]
7. Find the limit of each rational function as $x \to \infty$.

   a. \( \lim_{x \to \infty} \frac{3x + 7}{x^2 - 2} \)

   b. \( \lim_{x \to \infty} \frac{9x^4 + x}{2x^4 + 5x^2 + 1} \)

Extra Credit

Write down the equation of the line $f(x)$ that goes through the points $(2,6)$ and $(4,2)$. Then, write down a formal description (for example, an equation) for the function $g(x)$ that has a hole at $(3.5,3)$ but is otherwise equal to the function $f(x)$ for all points $x \neq 3.5$.

\[ f(x): \]

\[ g(x): \]