Solutions

Quiz 16: Sigma Notation

1. Write the sum without sigma notation. Then evaluate the sum.

\[
\sum_{k=1}^{3} (-1)^{k+1} \sin \left( \frac{\pi}{k} \right) = (-1)^{2} \sin \left( \frac{\pi}{1} \right) + (-1)^{3} \sin \left( \frac{\pi}{2} \right) + (-1)^{4} \sin \left( \frac{\pi}{3} \right)
\]

\[
= \sin \left( \pi \right) - \sin \left( \frac{\pi}{2} \right) + \sin \left( \frac{\pi}{3} \right)
\]

\[
= 0 - 1 + \frac{\sqrt{3}}{2}
\]

\[
= \frac{\sqrt{3}}{2} - 1
\]

2. Express the following sums in sigma notation. Depending on your choice of the lower limit of summation, there is more than one correct answer.

a. \[2 + 4 + 6 + 8 + 10 = \sum_{k=1}^{5} 2k\]

b. \[2 - 4 + 6 - 8 + 10 = \sum_{k=1}^{5} 2k(-1)^{k+1}\]

3. Evaluate the following three sums.

\[
\sum_{k=1}^{3} 7 = 7 + 7 + 7 = \boxed{21}
\]

\[
\sum_{k=1}^{11} 7 = \underbrace{7 + 7 + \ldots + 7}_{11 \text{ times}} = \boxed{77}
\]

\[
\sum_{k=3}^{11} 7 = \sum_{k=1}^{11} 7 - \sum_{k=1}^{3} 7 = 77 - 14 = \boxed{63}
\]