Example C: \( y = 5x^{2/5} - 2x \)

1) On the graph provided, label all maxima, minima and inflection points (max/min/I).

2) Find all critical points of \( y \):
   \[
y' = 2x^{-3/5} - 2 = 2(x^{-3/5} - 1) = 0
   \]
   \[
a \left( \frac{1}{x^{3/5}} - 1 \right) = 2 \left( \frac{1-x^{3/5}}{x^{3/5}} \right) = 0
   \]
   \[y' = 0 \text{ when } x = 1, \quad y' \text{ DNE when } x = 0 \]
   Critical points at \( x = 0, 1 \).

3) First derivative test:
   a. Identify the intervals where \( y \) is increasing or decreasing:
      \[
      \begin{array}{ccc}
      & - & + & - \\
      y' & & & \\
      0 & 1 & & \\
      \end{array}
      
      y \text{ increasing: } (0, 1) \\
      y \text{ decreasing: } (-\infty, 0), (1, \infty)
   
   b. What does the first derivative test tell us about each critical point?
      \( x = 0 \): \( f(0) \) is a min (since \( y' \) changes from - to +) \\
      \( x = 1 \): \( f(1) \) is a max (since \( y' \) changes from + to -)

4) Second derivative test:
   a. Identify the intervals where \( y \) is concave up or concave down:
      \[
      \begin{array}{ccc}
      & - & + & - \\
      y'' & & & \\
      0 & & & \\
      \end{array}
      
      y \text{ concave up: } \phi \\
      y \text{ concave down: } (-\infty, 0), (0, \infty)
   
   b. What does the second derivative test tell us about each critical point?
      \( x = 0 \): Test is inconclusive (since \( y'' \) DNE) \\
      \( x = 1 \): \( y(1) \) is a max (since \( y'' < 0 \))