1. Solve the following problem of Abu Kamil: Suppose 10 is divided into two parts and the product of one part by itself equals the product of the other part by the square root of 10. Find the parts.

Let one of the parts be \( x \), so that the other is \( 10 - x \). Then the given condition is that

\[
x^2 = (10 - x)\sqrt{10} \quad \Rightarrow \quad x^2 + \sqrt{10}x - 10\sqrt{10} = 0
\]

\[
\Rightarrow \quad x = \frac{-10 \pm \sqrt{100 - 40\sqrt{10}}}{2} = -5 \pm \sqrt{25 - 10\sqrt{10}}.
\]

2. Cite what you view as the major differences between Islamic mathematics and Greek mathematics. Ignore the obvious changes in emphasis from geometry to algebra. What about rigor? What about mathematical parochialism?

3. Show the essential step of Bombelli’s argument: if \( 2 + ib = \sqrt[3]{2 + 11i} \), then \( b = 1 \).

Cubing both sides gives

\[
8 + 12ib - 6b^2 - ib^3 = 2 + 11i \quad \Rightarrow \quad (6 - 6b^2) + i(12b - b^3 - 11) = 0.
\]

If \( \alpha + \beta i = 0 \), then \( \alpha = 0 \) and \( \beta = 0 \). Thus we have \( 6 - 6b^2 = 0 \) and \( 12b - b^3 - 11 = 0 \). From the first of these we see that \( b = \pm 1 \). The only one of these two values that also works for the second equation is \( b = 1 \), as required.

4. Show that trisecting the angle \( 60^\circ \) is equivalent to solving the cubic \( y^3 - 3y = 1 \). Then show that this cubic has no rational solutions.

For my typing convenience, we will drop the degree symbol, and just understand that all angle measures are in degrees.

Trisecting \( 60^\circ \) means finding \( 20^\circ \). This is equivalent to finding \( \cos(20) \).

Note that

\[
\frac{1}{2} = \cos(60) = \cos(40 + 20) = \cos(40)\cos(20) - \sin(40)\sin(20)
\]
\[
= (\cos^2(20) - \sin^2(20)) \cos(20) - 2 \sin(20) \cos(20) \sin(20)
\]
\[
= \cos^3(20) - 3 \sin^2(20) \cos(20) = \cos^3(20) - 3(1 - \cos^2(20)) \cos(20)
\]
\[
= 4 \cos^3(20) - 3 \cos(20).
\]

Multiplying everything by 2 gives
\[
8 \cos^3(20) - 6 \cos(20) = 1.
\]

If we set \( y = 2 \cos(20) \), we see that we are seeking \( y \) such that
\[
y^3 - 3y = 1.
\]

5. Derive a solution of Bombelli’s equation \( x^3 = 15x + 4 \) as a sum or difference of cube roots of imaginary numbers.

Substituting \( x = u + v \), we have
\[
u^3 + 3u^2v + 3uv^2 + v^3 = 15u + 15v + 4.
\]

Now assuming (as we may) that \( uv = 5 \), we may substitute \( v = 5/u \), obtaining
\[
u^3 + 15u + 75u^{-1} + 125u^{-3} = 15u + 75u^{-1} + 4
\]
\[
\Rightarrow u^6 - 4u^3 + 125 = 0 \quad \Rightarrow (u^3)^2 - 4u^3 + 125 = 0.
\]

This is a quadratic in \( u^3 \), so we have
\[
u^3 = \frac{4 \pm \sqrt{16 - 500}}{2} = 2 \pm \sqrt{-121}.
\]

Solving for \( v \) gives the same expression for \( v^3 \). Since \( u + v = x \) is real (because we know it equals 4), we must choose different signs for the imaginary parts of each, so that we have
\[
u^3 = 2 + \sqrt{-121} \quad \text{and} \quad v^3 = 2 - \sqrt{-121}.
\]

Thus we have
\[
x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}.
\]