Math 125 Review for Midterm 1 Spring 2016

1. The graph of the function $f$ is pictured below. Find the following limits or explain why they do not exist.

(a) $\lim_{x \to 3^+} f(x) = \cdots$  
(b) $\lim_{x \to 3^-} f(x) = \cdots$  
(c) $\lim_{x \to 3^+} f(x) = \cdots$  
(d) $\lim_{x \to 3^-} f(x) = \cdots$  
(e) $\lim_{x \to -3^+} f(x) = \cdots$  
(f) $\lim_{x \to -3^-} f(x) = \cdots$  
(g) List all numbers in the open interval $(-6, 6)$ at which $f$ is not continuous.

2. Complete the following definition of a limit:

Let $f(x)$ be defined on an open interval about $c$, except possibly at $c$ itself. We say that the limit of $f(x)$ at $x$ approaches $c$ is the number $L$, and write $\lim_{x \to c} f(x) = L$, if, for every $\epsilon > 0$, there exists a number $\delta > 0$ such that for all $x$,

3. Solve $\sin(2\theta) - \cos(\theta) = 0$ for the angle $\theta$, where $0 \leq \theta < 2\pi$. Show all your work. There might be multiple solutions.

4. Prove the limit statement $\lim_{x \to 3} \frac{4x - 7}{x - 3} = 5$. That is, for all $\epsilon > 0$, find $\delta > 0$ (depending on epsilon), such that, for all $x$ satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds.

4. Find each of the following limits. You must show all of your work to receive full credit.

(a) $\lim_{x \to 1^+} \frac{x - 1}{x - 1}$  
(b) $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$  
(c) $\lim_{h \to 0} \frac{1 - \sqrt{h + 1}}{h}$  
(d) $\lim_{x \to 3} \frac{2 - \sqrt{x^2 - 5}}{x + 3}$  
(e) $\lim_{\theta \to 0} \frac{1 + \theta + \sin \theta}{\cos \theta + 1}$

5. Given the function $\sqrt{2x^2 - 18}$, find all $c$ at which $f(x)$ is (a) continuous, (b) right-continuous but not continuous (c) left-continuous but not continuous.

6. Find each of the following limits. You must show all of your work to receive full credit.

(a) $\lim_{x \to 0} \frac{7x}{\tan \frac{5x}{2}}$  
(b) $\lim_{x \to -\infty} \frac{x^2 + 3x^3 - 16}{5x^3 + 9x^2 - x + 16}$  
(c) $\lim_{x \to \infty} \frac{\sqrt{x - 16} - \sqrt{x - 9}}{5x^2 + 16x - 4}$  
(d) $\lim_{x \to \infty} \frac{\sqrt{x^4 - 16}}{16x^2 - 4}$

7. Use the Intermediate Value Theorem to show that the equation $x^3 - 6x^2 + 8x + 1 = 0$ has three real solutions.