Rotation of a data vector by components

Given the attitude of a rotational axis and the amount and sense of rotation, generate general equations that calculate the new attitude of a data vector from its original attitude. All attitudes (orientations) are specified as azimuth and plunge, but are converted to directional components in the orthogonal coordinate system of:

positive X = due east, horizontal
positive Y = due north, horizontal
positive Z = vertical toward center of the earth

The magnitudes of any of the given vectors is of no consequence, only the attitude of the rotated vector is required. Input data has a blue background color, while answers have a light red background.

This worsheet is designed to solve structural geology problems that consist of rotations of orientation data. Input is in azimuth and plunge (decimal degrees).

$\text{Deg2Rad} := \dfrac{(\text{atan}(1.0)\cdot 4.0)}{180.0}$ $\quad\quad$ $\text{Deg2Rad} = 0.017$

Input the azimuth and plunge (decimal degrees) of the rotation axis below:

$\text{AxisAz} := 247$ $\quad$ $\text{AxisPl} := 8$

$a := \sin(\text{AxisAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $\quad$ $a = -0.912$

$b := \cos(\text{AxisAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $\quad$ $b = -0.387$

$c := \cos[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $\quad$ $c = 0.139$

a, b, and c are the directional components of the rotation axis

Input the azimuth and plunge (decimal degrees) of the data below:

$\text{DataAz} := 110$ $\quad$ $\text{DataPl} := 45$

$x := \sin(\text{DataAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $\quad$ $x = 0.664$

$y := \cos(\text{DataAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $\quad$ $y = -0.242$

$z := \cos[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $\quad$ $z = 0.707$

x, y, and z are the directional components of the data vector.

Input the amount and sense or rotation (degrees) below:

$\text{Rotation} := 270$

$r := \text{Rotation}\cdot\text{Deg2Rad}$ $\quad$ $r = 4.712$

r is the rotation value in radians; positive values rotate the data vector anticlockwise as viewed downplunge of the rotation axis.

The tprod() function simply calculates a quantity that is common to the rot_X(), rot_Y(), and rot_Z() functions below.

$\text{tprod}(x1, y1, z1, x2, y2, z2, r) := (x1\cdot x2 + y1\cdot y2 + z1\cdot z2)\cdot(1 - \cos(r))$

The functions rot_X(), rot_Y(), and rot_Z() return the directional components of the rotated data vector given the components of the original data vector (x1,y1,z1), the components of the rotation axis (x2,y2,z2), and the amount or rotation (r). The "tp" argument is calculated by the tprod() function.

$\text{rot\_X}(x1, y1, z1, x2, y2, z2, r, tp) := \cos(r)\cdot x1 + tp\cdot x2 + [\sin(r)\cdot(y2\cdot z1 - z2\cdot y1)]$

$\text{rot\_Y}(x1, y1, z1, x2, y2, z2, r, tp) := \cos(r)\cdot y1 + tp\cdot y2 - [\sin(r)\cdot(x2\cdot z1 - z2\cdot x1)]$

$\text{rot\_Z}(x1, y1, z1, x2, y2, z2, r, tp) := \cos(r)\cdot z1 + tp\cdot z2 + [\sin(r)\cdot(x2\cdot y1 - y2\cdot x1)]$

$t1 := tprod(x, y, z, a, b, c, r)$

$R\_X := rot\_X(x, y, z, a, b, c, r, t1)$ $R\_X = 0.617$

$R\_Y := rot\_Y(x, y, z, a, b, c, r, t1)$ $R\_Y = -0.577$

$R\_Z := rot\_Z(x, y, z, a, b, c, r, t1)$ $R\_Z = -0.535$

R_X, R_Y, and R_Z are the directional components of the rotated data vector.

$$RD := \begin{cases} \begin{pmatrix} R\_X \\ R\_Y \\ R\_Z \end{pmatrix} & \text{if } R\_Z \geq 0.0 \\ \begin{pmatrix} -R\_X \\ -R\_Y \\ -R\_Z \end{pmatrix} & \text{otherwise} \end{cases}$$

$$RD = \begin{pmatrix} -0.617 \\ 0.577 \\ 0.535 \end{pmatrix}$$

If R_Z < 0.0, the rotated vector plots in the upper hemisphere and must be reflected back to the lower hemisphere.

Validity check:

$$(RD_0)^2 + (RD_1)^2 + (RD_2)^2 = 1$$

The sum of squares of the directional components must equal 1.0 for a valid vector.

$$\alpha := \frac{acos(RD_0)}{Deg2Rad} \qquad \alpha = 128.101$$

α, β, and γ are the directional angles of the rotated data vector in degrees.

$$\beta := \frac{acos(RD_1)}{Deg2Rad} \qquad \beta = 54.763$$

$$\gamma := \frac{acos(RD_2)}{Deg2Rad} \qquad \gamma = 57.647$$

$$azimuth := \begin{cases} 450 - \frac{atan2(RD_0, RD_1)}{Deg2Rad} & \text{if } RD_0 < 0.0 \wedge RD_1 \geq 0.0 \\ 90 - \frac{atan2(RD_0, RD_1)}{Deg2Rad} & \text{otherwise} \end{cases}$$

$azimuth = 313.077$

The azimuth of the rotated vector in degrees.

$$plunge := 90 - \frac{acos(RD_2)}{Deg2Rad}$$

$plunge = 32.353$

The plunge of the rotated vector in degrees.

Graphical Plot

The below calculations are used to plot an equal-area or equal-angle lower-hemisphere stereographic projection plot of the position of the original data vector, the rotation axis, the position of the rotated data vector, and the path of the rotation.

"t" is a counter that increments the number of steps used to trace the rotation path. "x_cent", "y_cent", and "radius_size" are the center of the projection and radius size of the projection respectively.

$\text{tmax} := 360$ The "tmax" varible controls the number of steps used to draw the rotation path.

$t := 0, 1 .. \text{tmax}$ $\text{x\_cent} := 0.0$ $\text{y\_cent} := 0.0$ $\text{radius\_size} := 3.75$

The functions cx(t) and cy(t) trace the outline (primitive) of the stereographic projection.

$$cx(t) := \left(\text{x\_cent} + \cos\left(360 \cdot \frac{t}{\text{tmax}} \text{Deg2Rad}\right) \cdot \text{radius\_size}\right)$$

$$cy(t) := \left(\text{y\_cent} + \sin\left(360 \frac{t}{\text{tmax}} \cdot \text{Deg2Rad}\right) \cdot \text{radius\_size}\right)$$ $\text{alArea} := 0$ $\text{EqualAngle} := 1$

Set the variable "projection_flag" equal to "EqualArea" or "EqualAngle" to control the type of projection for the graphical plot.

$\text{projection\_flag} := \text{EqualArea}$

The below sections calculate the polar coordinates of the original data, the rotation axis, and the rotated data vector position based on the above calculated solution. Variables ending in "_mag" are the magnitude of the plotted position relative to the center of the diagram. The azimuth is already calculated above. The x,y position of the elements of the problem are stored in the variables ending with "_x" and "_y".

$$\text{data\_mag} := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\frac{\text{acos}(|z|)}{2}\right)\right) \cdot \text{radius\_size} & \text{if projection\_flag} = \text{EqualArea} \\ \tan\left(\frac{\text{acos}(z)}{2}\right) \cdot \text{radius\_size} & \text{otherwise} \end{cases}$$

$\text{data\_x} := \text{x\_cent} + \sin(\text{DataAz} \cdot \text{Deg2Rad}) \cdot \text{data\_mag}$ $\text{data\_x} = 1.907$

$\text{data\_y} := \text{y\_cent} + \cos(\text{DataAz} \cdot \text{Deg2Rad}) \cdot \text{data\_mag}$ $\text{data\_y} = -0.694$

$$\text{axis\_mag} := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\frac{\text{acos}(|c|)}{2}\right)\right) \cdot \text{radius\_size} & \text{if projection\_flag} = \text{EqualArea} \\ \tan\left(\frac{\text{acos}(c)}{2}\right) \cdot \text{radius\_size} & \text{otherwise} \end{cases}$$

$\text{axis\_x} := \text{x\_cent} + \sin(\text{AxisAz} \cdot \text{Deg2Rad}) \cdot \text{axis\_mag}$ $\text{axis\_x} = -3.203$

$\text{axis\_y} := \text{y\_cent} + \cos(\text{AxisAz} \cdot \text{Deg2Rad}) \cdot \text{axis\_mag}$ $\text{axis\_y} = -1.359$

$$\text{rot\_mag} := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\frac{\text{acos}(|\text{R\_Z}|)}{2}\right)\right) \cdot \text{radius\_size} & \text{if projection\_flag} = \text{EqualArea} \\ \tan\left(\frac{\text{acos}(\text{R\_Z})}{2}\right) \cdot \text{radius\_size} & \text{otherwise} \end{cases}$$

$\text{rotated\_x} := \text{x\_cent} + \sin(\text{azimuth} \cdot \text{Deg2Rad}) \cdot \text{rot\_mag}$ $\text{rotated\_x} = -1.868$

$\text{rotated\_y} := \text{y\_cent} + \cos(\text{azimuth} \cdot \text{Deg2Rad}) \cdot \text{rot\_mag}$ $\text{rotated\_y} = 1.746$

The function rot(t) returns the rotation angle as a function of the iteration variable "t" that steps through 0..180 steps. Thus the total rotation is divided into 180 steps to trace the rotation path.

$$rot(t) := \frac{t}{tmax} \cdot Rotation \cdot Deg2Rad$$

The function tdot(t) returns a quantity needed for the calculation of rx(), ry), and rz() as a function of the rotation stepping variable "t". The flag(t) variable flags whether or not the rotated vector plots in the lower hemisphere- if it is positive it plots in the lower hemisphere.

$$tdot(t) := tprod(x, y, z, a, b, c, rot(t))$$

$$flag(t) := sign(rot\_Z(x, y, z, a, b, c, rot(t), tdot(t)))$$

The functions rx(t), ry(t), and rz(t) return the directional cosines of the rotated data vector position as a function of the stepping variable "t". The components are adjusted to always plot in the lower hemisphere of the stereographic plot.

$$rx(t) := rot\_X(x, y, z, a, b, c, rot(t), tdot(t)) \cdot flag(t)$$

$$ry(t) := rot\_Y(x, y, z, a, b, c, rot(t), tdot(t)) \cdot flag(t)$$

$$rz(t) := rot\_Z(x, y, z, a, b, c, rot(t), tdot(t)) \cdot flag(t)$$

The functions rotaz(t) and rotmag(t) return the polar coordinates of the rotation path on the graphical plot as a function of the stepping variable "t". Both functions return values as degrees.

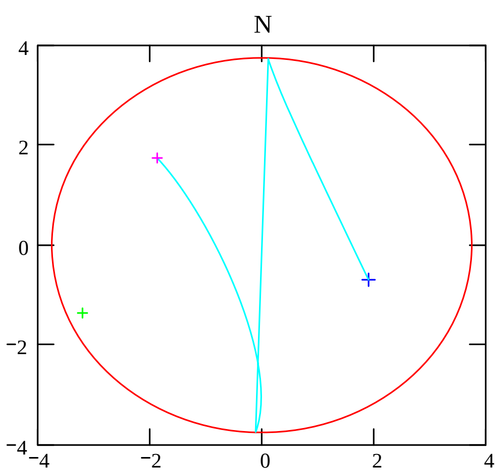
$$rotaz(t) := \begin{cases} 450 - \dfrac{atan2(rx(t), ry(t))}{Deg2Rad} & \text{if } rx(t) < 0.0 \wedge ry(t) \geq 0.0 \\ 90 - \dfrac{atan2(rx(t), ry(t))}{Deg2Rad} & \text{otherwise} \end{cases}$$

$$rotmag(t) := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\dfrac{acos(|rz(t)|)}{2} \right) \right) \cdot radius\_size & \text{if } projection\_flag = 0 \\ \tan\left(\dfrac{acos(|rz(t)|)}{2} \right) \cdot radius\_size & \text{otherwise} \end{cases}$$

The functions pl_x(t) and pl_y(t) return the x,y coordinates of the rotation path on the graphical diagram as a function of the "t" stepping variable.

$$pl\_x(t) := x\_cent + \sin(rotaz(t) \cdot Deg2Rad) \cdot rotmag(t)$$

$$pl\_y(t) := y\_cent + \cos(rotaz(t) \cdot Deg2Rad) \cdot rotmag(t)$$



Original data: blue cross
Axis: green cross
Rotated: magenta cross