Rotation of a data vector about an axis to a new attitude

Given the attitude of a rotational axis and the amount and sense of rotation, generate general equations that calculate the new attitude of a data vector from its original attitude. All attitudes (orientations) are specified as azimuth and plunge, but are converted to directional components in the orthogonal coordinate system of:

positive X = due east, horizontal
positive Y = due north, horizontal
positive Z = vertical toward center of the earth

The magnitudes of any of the given vectors is of no consequence, only the attitude of the rotated vector is required. Input data has a blue background, answers have a light red background.

$\text{Deg2Rad} := \dfrac{(\text{atan}(1.0)\cdot 4.0)}{180.0}$ $\text{Deg2Rad} = 0.017$

Deg2Rad is simply the conversion factor for degrees to radians.

$\text{AxisAz} := 247$ $\text{AxisPl} := 8$

$a := \sin(\text{AxisAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $a = -0.912$

$b := \cos(\text{AxisAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $b = -0.387$

$c := \cos[\text{Deg2Rad}\cdot(90 - \text{AxisPl})]$ $c = 0.139$

a, b, and c are the directional components of the rotation axis calculated from the given azimuth and plunge.

$\text{DataAz} := 110$ $\text{DataPl} := 45$

$x := \sin(\text{DataAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $x = 0.664$

$y := \cos(\text{DataAz}\cdot\text{Deg2Rad})\cdot\sin[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $y = -0.242$

$z := \cos[\text{Deg2Rad}\cdot(90 - \text{DataPl})]$ $z = 0.707$

x, y and z are the directional components of the data vector calculated from the given azimuth and plunge.

$\text{Rotation} := 330$

$r := \text{Rotation}\cdot\text{Deg2Rad}$ $r = 5.76$

r is the rotation amount and sense, in radians. Positive rotation is anticlockwise as viewed down the plunge of the rotation axis.

Validity check:

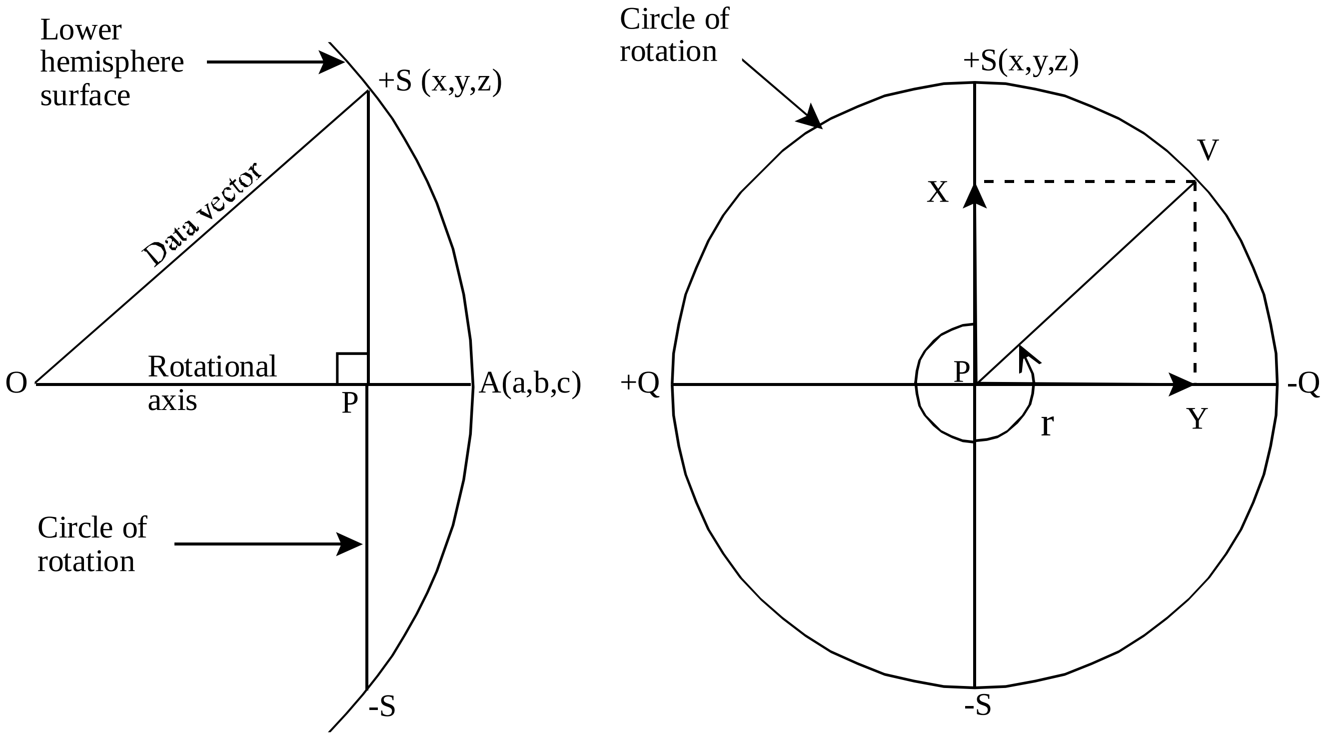
$a^2 + b^2 + c^2 = 1$

$x^2 + y^2 + z^2 = 1$

$OA := \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $OS := \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Vector OA is the rotation axis, OS is the pre-rotation attitude of the data vector.

Geometry of the Rotation of a Data Vector by Vector Addition



$OP := \overrightarrow{OA} \cdot \left(\left(\overrightarrow{OA} \cdot \overrightarrow{OS}\right)\right)$

OP is the rotational axis multiplied by the dot product of the rotation axis and data vector. This yields the vector with head at the center of the circle of rotation (OP).

$PQ := \overrightarrow{OA} \times \overrightarrow{OS}$

$PS := \overrightarrow{PQ} \times \overrightarrow{OA}$

PQ is the vector perpindicular to the cross product of OA and OS. The magnitude of the cross product is equal to (OA)(OS)(sin t) where t is the angle between OA and OS. Since OA and OS are unity, PQ is exactly the magnitude to "touch" the circle of rotation. PS is then calculated by taking the cross product of PQ and OA, yielding a vector as depicted above.

$PX := \cos(r) \cdot \overrightarrow{PS}$

PX is the projection of the rotated data vector (PV) upon the PS vector.

$PY := \sin(r) \cdot \overrightarrow{PQ}$

PY is the projection of the rotated data vector (PV) upon the PQ vector.

$\vec{OV} := \vec{OP} + \vec{PX} + \vec{PY}$ By adding OP, PX, and PY "head-to-tail", the rotated data vector is calculated in terms of the orthogonal coordinate system defined above.

$OV = \begin{pmatrix} 0.746 \\ -0.557 \\ 0.366 \end{pmatrix}$

$OV := \begin{cases} OV & \text{if } OV_2 > 0.0 \\ -OV & \text{otherwise} \end{cases}$

Validity check:

$(OV_0)^2 + (OV_1)^2 + (OV_2)^2 = 1$

$\alpha := \frac{acos(OV_0)}{Deg2Rad}$ $\alpha = 41.76$

$\beta := \frac{acos(OV_1)}{Deg2Rad}$ $\beta = 123.815$

$\gamma := \frac{acos(OV_2)}{Deg2Rad}$ $\gamma = 68.538$

$azimuth := \begin{cases} 450 - \frac{atan2(OV_0, OV_1)}{Deg2Rad} & \text{if } (OV_0 < 0) \wedge (OV_1 \geq 0) \\ 90 - \frac{atan2(OV_0, OV_1)}{Deg2Rad} & \text{otherwise} \end{cases}$ $azimuth = 126.725$

$plunge := (90 - \gamma)$ $plunge = 21.462$

Graphical Plot

The "tmax" varible controls the number of steps used to draw the rotation path.

$tmax := 360$

$t := 0, 1 .. tmax$

$rot(t) := r \cdot \frac{t}{tmax}$

$x\_cent := 0.0$ $y\_cent := 0.0$ $radius\_size := 3.75$

$EqualArea := 0$ $EqualAngle := 1$

Set the variable "projection_flag" equal to "EqualArea" or "EqualAngle" to control the type of projection for the graphical plot.

$pl\_OV(t) := \vec{OP} + \cos(rot(t)) \cdot \vec{PS} + \sin(rot(t)) \cdot \vec{PQ}$

$projection\_flag := EqualAngle$

The functions rx(), ry(), and rz() return the directional components of a rotated vector as a function of the rotation stepping variable "t". The result of each function is multiplied by the sign of the z component to reflect the result to the lower hemisphere projection.

$rx(t) := pl\_OV(t)_0 \cdot sign(pl\_OV(t)_2)$

$ry(t) := pl\_OV(t)_1 \cdot sign(pl\_OV(t)_2)$

$rz(t) := pl\_OV(t)_2 \cdot sign(pl\_OV(t)_2)$

The functions cx(t) and cy(t) trace the outline (primitive) of the stereographic projection.

$$rotaz(t) := \begin{cases} 450 - \dfrac{atan2(rx(t), ry(t))}{Deg2Rad} & \text{if } (rx(t) < 0) \wedge (ry(t) \geq 0) \\ 90 - \dfrac{atan2(rx(t), ry(t))}{Deg2Rad} & \text{otherwise} \end{cases}$$

$$cx(t) := \left(x\_cent + \cos\left(360 \cdot \frac{t}{tmax} Deg2Rad\right) \cdot radius\_size\right)$$

$$cy(t) := \left(y\_cent + \sin\left(360 \frac{t}{tmax} \cdot Deg2Rad\right) \cdot radius\_size\right)$$

$$rotmag(t) := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\dfrac{acos(|rz(t)|)}{2}\right)\right) \cdot radius\_size & \text{if } projection\_flag = 0 \\ \tan\left(\dfrac{acos(|rz(t)|)}{2}\right) \cdot radius\_size & \text{otherwise} \end{cases}$$

The functions rotaz() and rotmag() return the azimuth of the rotated vector and the magnitude of the vector as a function of the rotation angle stepping variable "t".

The functions pl_x(t) and pl_y(t) return the x,y coordinates of the rotation path on the graphical diagram as a function of the "t" stepping variable.

$$pl\_x(t) := x\_cent + \sin(rotaz(t) \cdot Deg2Rad) \cdot rotmag(t)$$

$$pl\_y(t) := y\_cent + \cos(rotaz(t) \cdot Deg2Rad) \cdot rotmag(t)$$

The data_mag, axis_mag, and rot_mag results contain the magnitude of the original, rotation axis, and roated vector as plotted on the stereographic projection. Likewise (data_x,data_y), (axis_x,axis_y), and (rotated_x,rotated_y) are the (x,y) coordinates of the respective vectors on the projection diagram.

$$data\_mag := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\dfrac{acos(|z|)}{2}\right)\right) \cdot radius\_size & \text{if } projection\_flag = EqualArea \\ \tan\left(\dfrac{acos(z)}{2}\right) \cdot radius\_size & \text{otherwise} \end{cases}$$

$$rot\_mag := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\dfrac{acos(|OV_2|)}{2}\right)\right) \cdot radius\_size & \text{if } projection\_flag = EqualArea \\ \tan\left(\dfrac{acos(OV_2)}{2}\right) \cdot radius\_size & \text{otherwise} \end{cases}$$

$$data\_x := x\_cent + \sin(DataAz \cdot Deg2Rad) \cdot data\_mag \qquad data\_x = 1.46$$

$$data\_y := y\_cent + \cos(DataAz \cdot Deg2Rad) \cdot data\_mag \qquad data\_y = -0.531$$

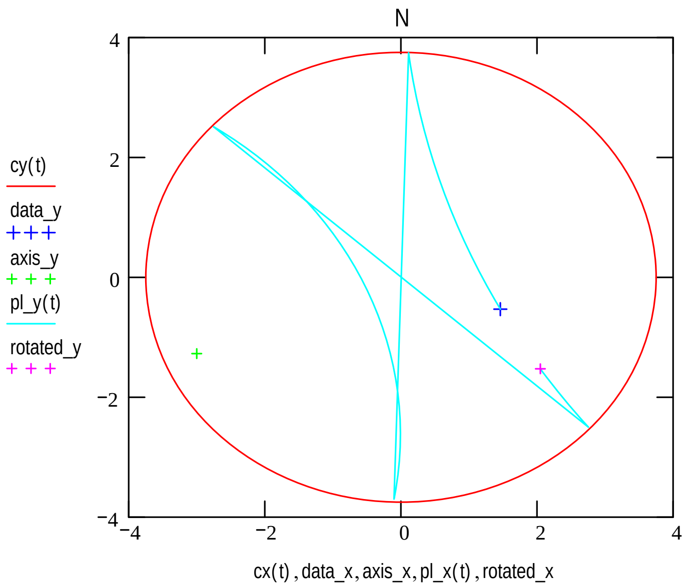
$$rotated\_x := x\_cent + \sin(azimuth \cdot Deg2Rad) \cdot rot\_mag \qquad rotated\_x = 2.048$$

$$rotated\_y := y\_cent + \cos(azimuth \cdot Deg2Rad) \cdot rot\_mag \qquad rotated\_y = -1.528$$

$$axis\_mag := \begin{cases} \sqrt{2} \cdot \left(\sin\left(\dfrac{acos(|c|)}{2}\right)\right) \cdot radius\_size & \text{if } projection\_flag = EqualArea \\ \tan\left(\dfrac{acos(c)}{2}\right) \cdot radius\_size & \text{otherwise} \end{cases}$$

$axis\_x := x\_cent + \sin(AxisAz \cdot Deg2Rad) \cdot axis\_mag$ $axis\_x = -3.001$

$axis\_y := y\_cent + \cos(AxisAz \cdot Deg2Rad) \cdot axis\_mag$ $axis\_y = -1.274$



Original data: blue cross
Axis: green cross
Rotated: magenta cross