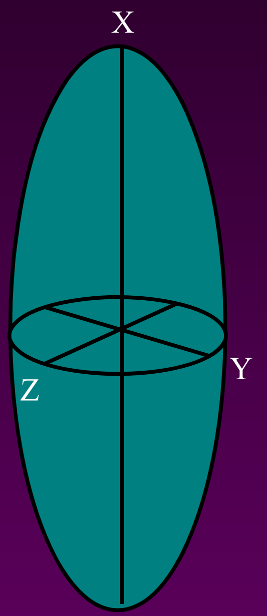
GY403 Structural Geology

The general equations of the Mohr Circle for strain

Strain Ellipsoid

A three-dimensional ellipsoid that describes the magnitude of dilational and distortional strain

- Assume a perfect sphere before deformation
- Three mutually perpendicular axes X, Y, and Z
- X is maximum stretch (S_X) and Z is minimum stretch (S_Z)
- There are unique directions corresponding to values of S_X and S_Z, but an infinite number of directions corresponding to S_Y



Strain

The results of deformation via distortion and dilation

- Heterogeneous strain: strain ellipsoid varies from point-to-point in deformed body
- Homogenous strain: strain ellipsoid is equivalent from point-to-point in deformed body
- Although hetereogenous strain is the rule in real rocks, often portions of a deformed body behave as homogenous with respect to strain

Homogeneous Strain “Ground Rules”

Characteristics of homogenous strain

- Straight lines that exist in the non-rigid body remain straight after deformation
- Lines that are parallel in the non-rigid body remain parallel after deformation
- In a special case of homogenous strain termed “Plane Strain”, volume and area are conserved

General Strain Equations

Extension (e), Stretch (S), and Quadratic Elongation (λ)

These equations measure linear strain :

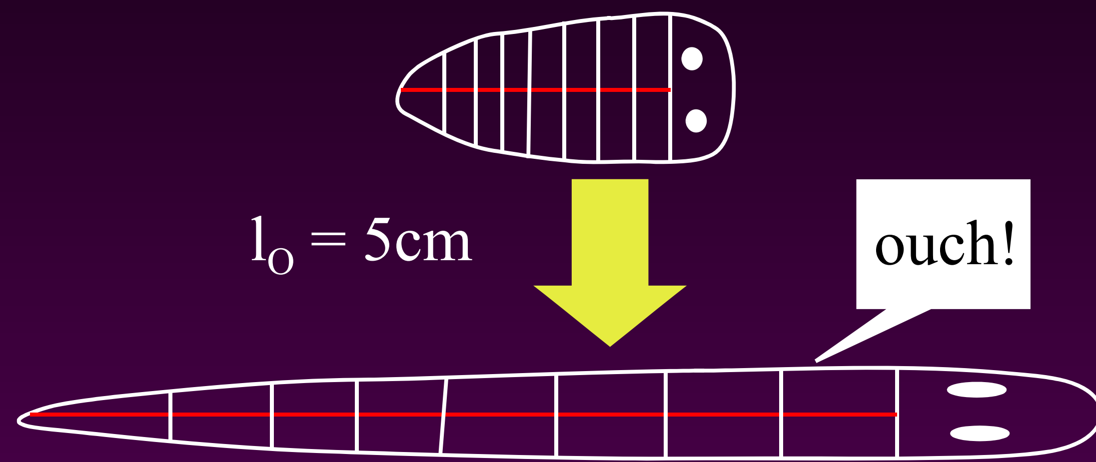
l_O = original length

l_F = final length

$$e = \frac{l_F - l_O}{l_O}$$

$$S = \frac{l_F}{l_O}$$

$$\lambda = \left[\frac{l_F}{l_O}\right]^2$$



l_O = 5cm

l_F = 12cm

$$S = \frac{l_F}{l_O} = \frac{12cm}{5cm} = 2.4$$

$e = (S-1) = 2.4 - 1 = 1.4$

$\lambda = S^2 = (2.4)^2 = 5.76$

Rotational Strain Equations

quantifying angular shear (ψ) and shear strain (γ)

θ = angle between reference line (L) and maximum stretch (X) measured from X to A (clockwise=+; anticlockwise=-)

ψ_L (perpendicular to L relative to M) = -40

$\gamma_L = \tan(\psi_L) = \tan(-40) = -0.839$

$\alpha_L = \theta_d - \theta = (-25) - (-35) = +10$
angle of internal rotation

Mohr Circle for Strain

General equations as a function of λ_X, λ_Z, and θ_d

$$\lambda' = \frac{1}{\lambda}$$

λ_X = quadratic elongation parallel to X axis of finite strain ellipse

λ_Z = quadratic elongation parallel to Z axis of finite strain ellipse

$$\lambda' = \frac{\lambda'_Z + \lambda'_X}{2} - \frac{\lambda'_Z - \lambda'_X}{2} \cos(2\theta_d)$$

$$\frac{\gamma}{\lambda} = \frac{\lambda'_Z - \lambda'_X}{2} \sin(2\theta_d)$$

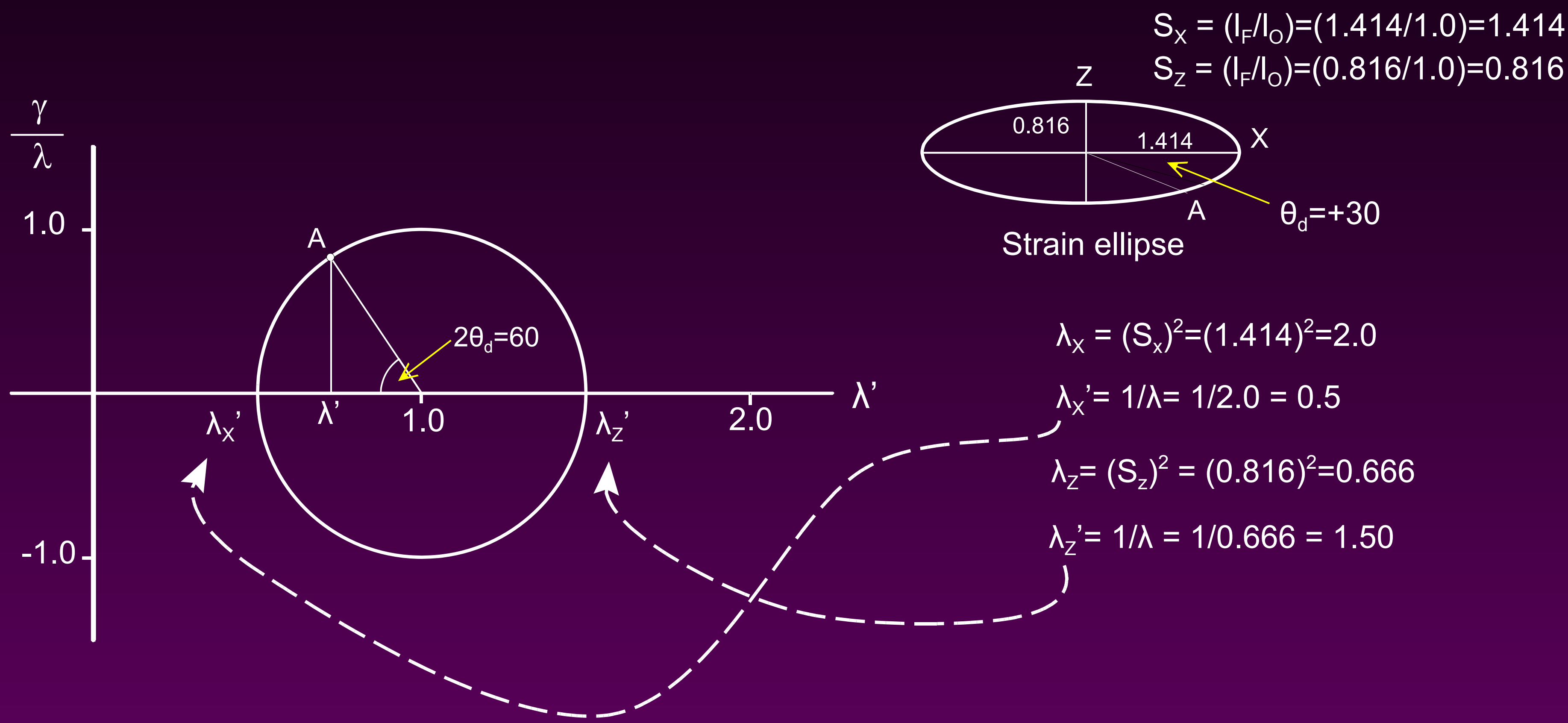
$$\tan \theta_d = \tan \theta \, \frac{S_Z}{S_X}$$

$$\alpha = \theta_d - \theta$$

(internal rotation)

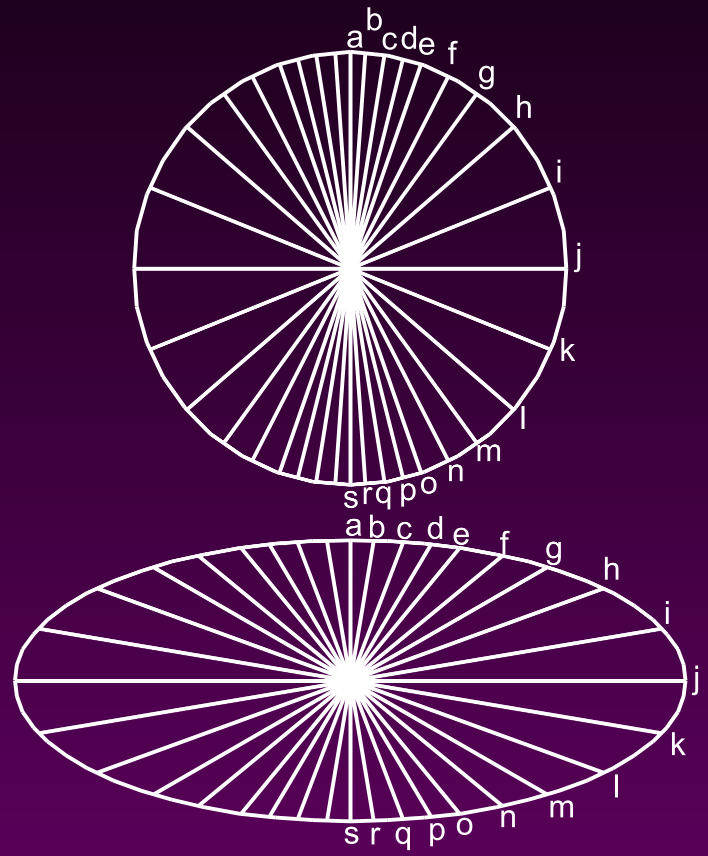
Mohr Circle for Strain

Geometric relations between the finite strain ellipse and the Mohr Circle for strain



Mohr Circle for Strain

Reference lines in the undeformed and deformed state



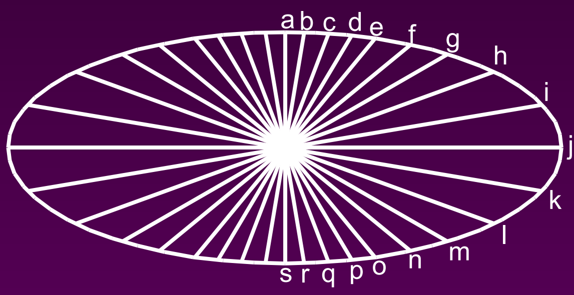
S_X=1.936
S_Z=0.707

Mohr Circle Strain Relationships

Values of quadratic elongation (λ), shear strain (γ), original θ angle, angular shear (ψ), and angle of internal rotation (α) as a function of θ_d

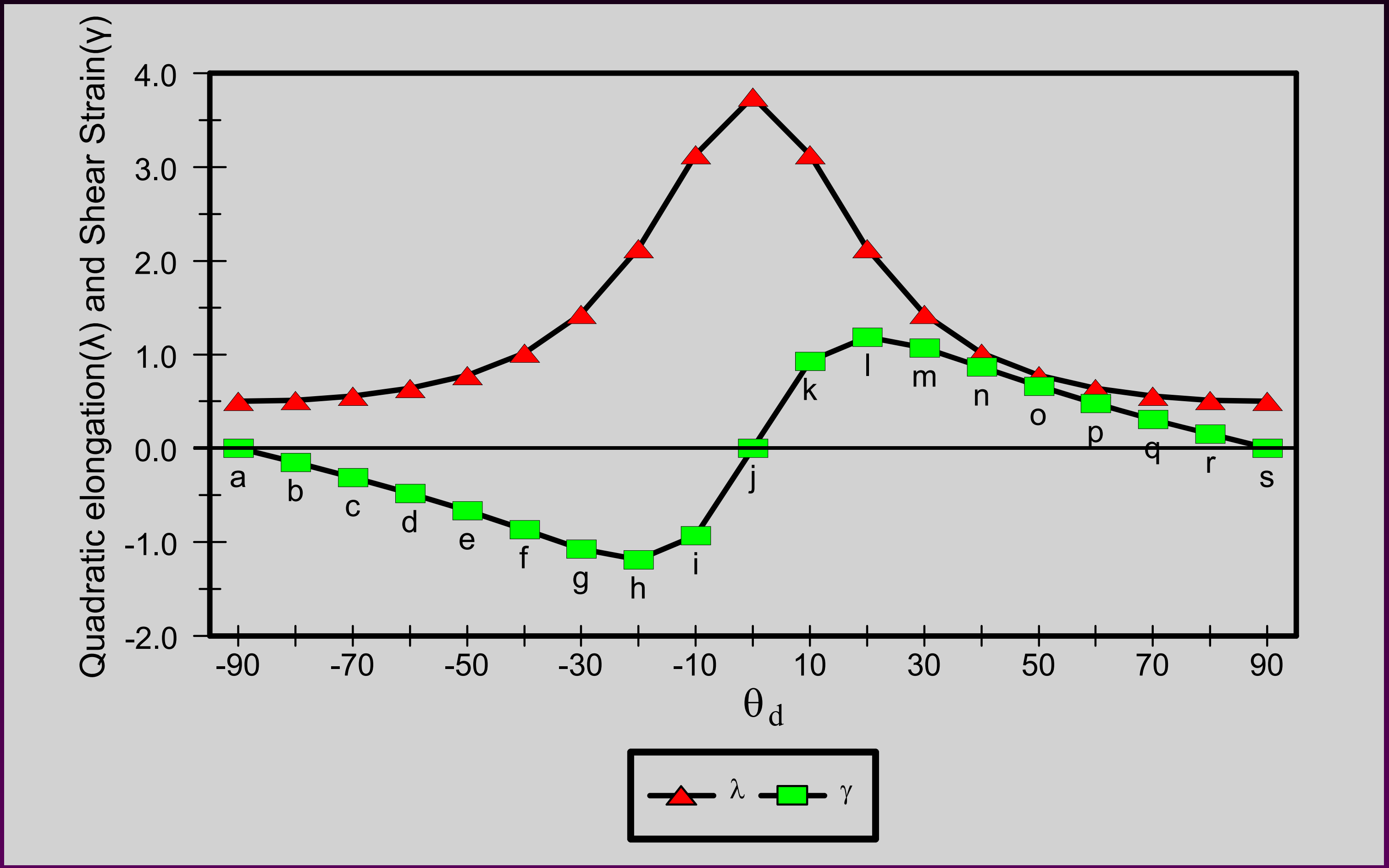
Line	θ_d	λ	γ	θ	ψ	α
a	-90	0.500	-0.000	-90.0	-0.0	0.0
b	-80	0.513	-0.152	-86.3	-8.7	6.3
c	-70	0.556	-0.310	-82.4	-17.2	12.4
d	-60	0.638	-0.479	-78.1	-25.6	18.1
e	-50	0.779	-0.665	-73.0	-33.6	23.0
f	-40	1.017	-0.868	-66.5	-41.0	26.5
g	-30	1.428	-1.072	-57.7	-47.0	27.7
h	-20	2.129	-1.187	-44.9	-49.9	24.9
i	-10	3.134	-0.929	-25.8	-42.9	15.8
j	0	3.748	0.000	0.0	0.0	0.0
k	10	3.134	0.929	25.8	42.9	-15.8
l	20	2.129	1.187	44.9	49.9	-24.9
m	30	1.428	1.072	57.7	47.0	-27.7
n	40	1.017	0.868	66.5	41.0	-26.5
o	50	0.779	0.665	73.0	33.6	-23.0
p	60	0.638	0.479	78.1	25.6	-18.1
q	70	0.556	0.310	82.4	17.2	-12.4
r	80	0.513	0.152	86.3	8.7	-6.3
s	90	0.500	0.000	90.0	0.0	0.0

S_X=1.936
S_Z=0.707



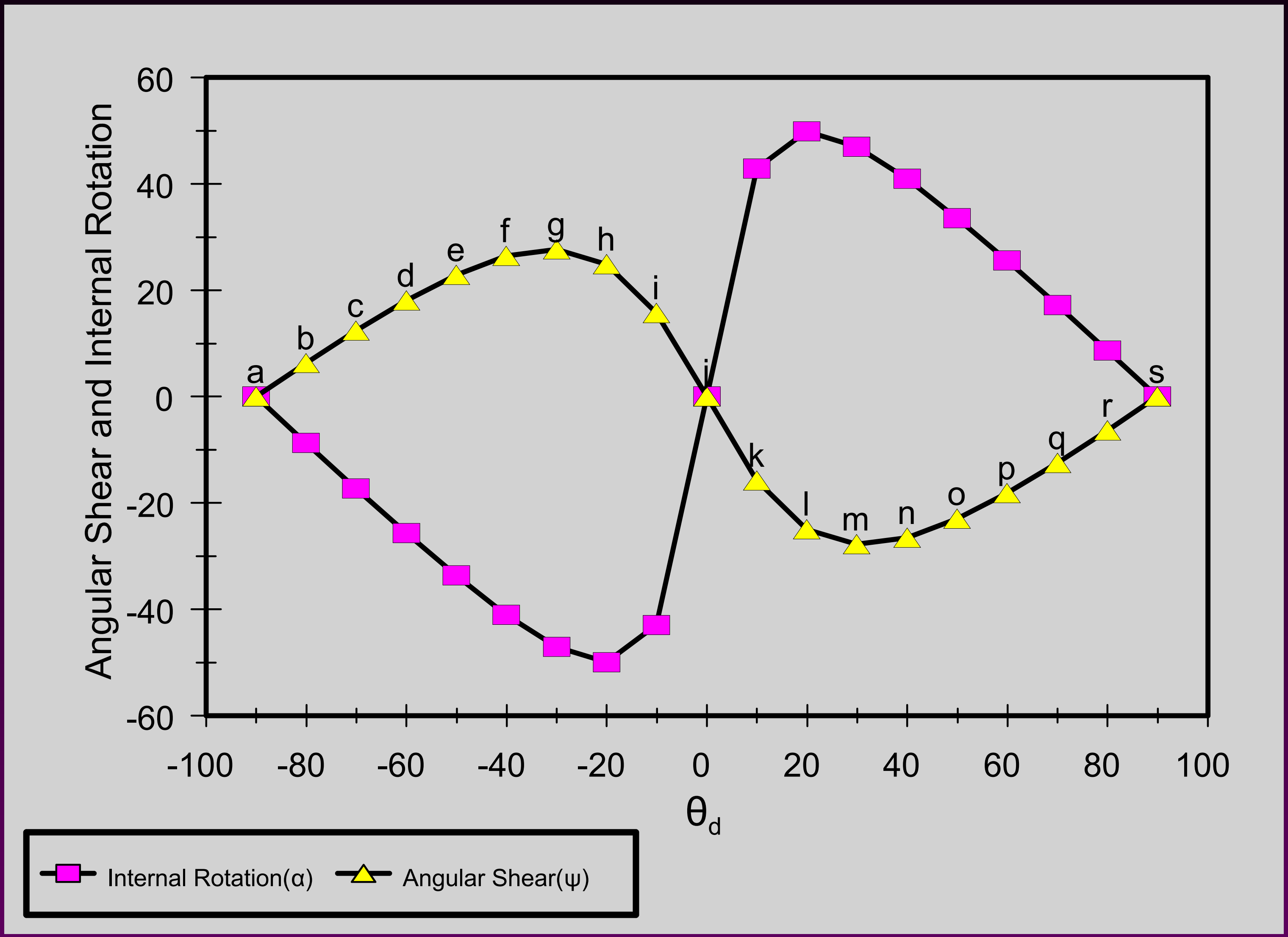
Strain Ellipse General Equation

Values for quadratic elongation (λ) and shear strain (γ) as a function of θ_d



Strain Ellipse General Equation

Values for angular shear (ψ) and internal rotation (α) as a function of θ_d



Example strain problem

Given a finite strain ellipse of S_X=1.936 and S_Z=0.707, find for direction θ_d=-20° values of S, λ, γ, ψ, and α

$\lambda_X=(1.936)^2 = 3.750$; $\lambda_Z = (0.707)^2 = 0.500$; $\lambda'_X=0.267$; $\lambda'_Z=2.0$

$$\lambda' = \frac{2.0+0.267}{2} - \frac{2.0 - 0.267}{2} \text{Cos}(-40) = 1.133-(0.866)(0.766) = 0.470$$

$\lambda = 1/\lambda' = 1/0.470 = 2.128 \quad \therefore S = (2.128)^{0.5} = 1.459$

$$\gamma = \frac{2.0-0.267}{2} \text{Sin}(-40)\,\lambda = (0.866)(-0.643)(2.128) = -1.185$$

$\psi = \tan^{-1}(\gamma) = \tan^{-1}(-1.185) = -49.8°$

$$\text{Tan}(\theta_d) = \tan(\theta)\frac{S_Z}{S_X} \quad \therefore \quad \text{Tan}(\theta) = \tan(\theta_d)\frac{S_X}{S_Z} \quad \therefore \quad \theta = -44.9°$$

$\alpha = \theta_d - \theta = (-20) - (-44.9) = +24.9°$

Application of Plane Strain

Deformed oolids from the study of Cloos (1947)

Assuming plane strain: no dilational component to strain, therefore, constant volume applies:

$V_S = 4/3\pi r^3$ where r is the radius of the sphere

$V_e = 4/3\pi abc$ where (a,b,c) are the ½ axial legths of the ellipsoid

$V_s = V_e$

$4/3\pi r^3 = 4/3\pi abc$

Because of plane strain $r = b \therefore$

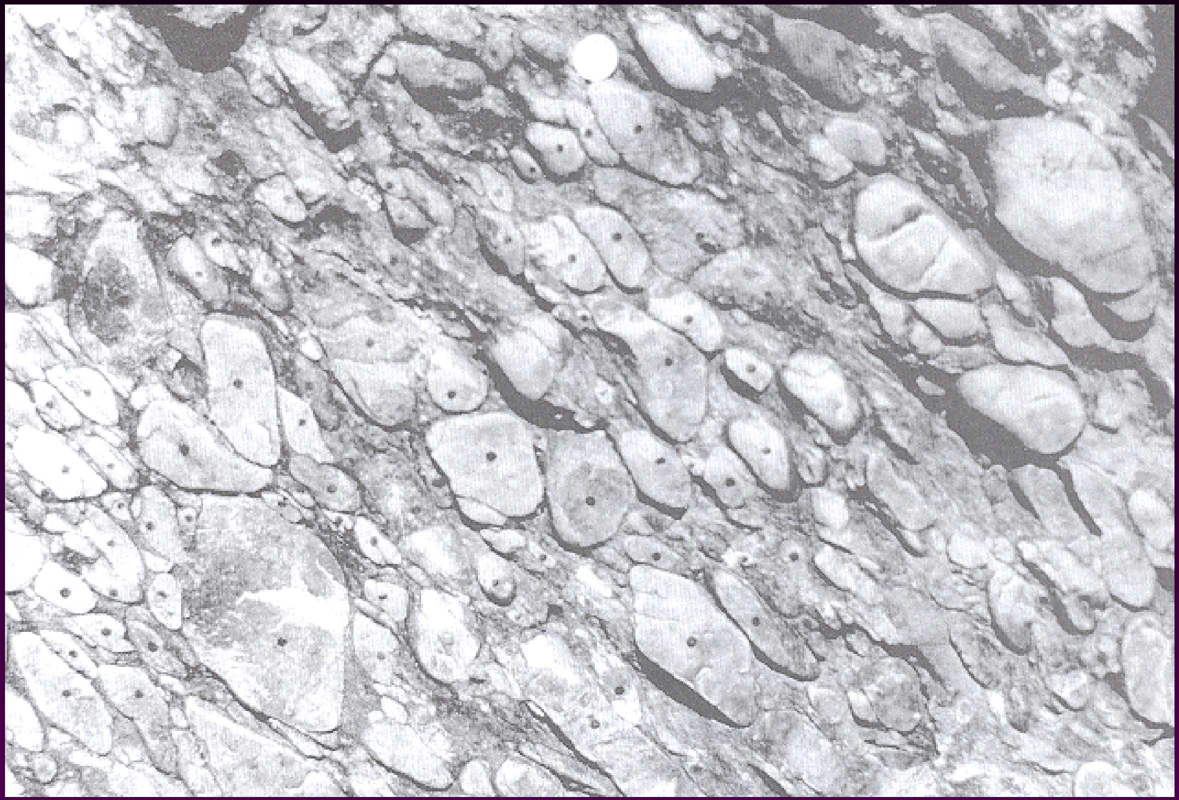
$r^2 = ac$

$r = (ac)^{0.5}$

Example: a=4.2mm; c=2.5mm; $r=(4.2*2.5)^{0.5} = 3.3 \therefore S_x = 4.2/3.3 = 1.27$

Application to Deformed Strain Markers

- Markers may be original spheres or ellipsoids
- Pebbles, sand grains, reduction spots, ooids, fossils, etc.
- Assume homogenous strain domain



Measuring Length/Width Ratios (R_f)

- Measure major and minor axis of each strain ellipse
- Rf = (Major/minor) (yields a unitless ratio)
- φ = Angle from reference direction (usually foliation or cleavage), positive angles are clockwise, negative counterclockwise

Spreadsheet setup for Rf/ φ analysis

Ellipse	Length	Width	Rf	ϕ
1	0.3066	0.1600	1.916	32.8
2	0.0969	0.0704	1.376	51.9
3	0.1221	0.0729	1.675	61.8
4	0.0660	0.0389	1.697	54.6
5	0.1735	0.1392	1.246	22.7
6	0.0825	0.0539	1.531	76.2
7	0.1770	0.1275	1.388	67.5
8	0.0736	0.0347	2.121	37.6
9	0.0937	0.0797	1.176	-0.3
10	0.1184	0.0457	2.591	10.6

- Note: φ is measured relative to a chosen reference direction such as foliation

Hyperbolic Net

- Used to plot strain markers that were originally ellipsoidal
- Statistically the Rf ratios will tend to fall along one of the hyperbolic curves