Examples of non-orientable surfaces in 4-dimensional space

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Intro

1. Thanks to Professor Bae and KNU students
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2. Joint work with Sera Kim
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3. Two examples of unknotted spheres in 4-space
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4. Movies, charts, and movie moves
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5. The standard cross-cap
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6. A non-standard view
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12. Famous problems
• Kamada’s surface braids
• Black vertices and the associated movie
• closure
Positive Cross Cap

Negative Cross Cap
Normal Euler Class

\[ \text{Lk} = 1 \]
$E \in \{-2n, -2n+2, \ldots, 2n-3, 2n\}$
Whitney’s conjecture/Massey’s Theorem

The normal Euler class $E$ of a non-orientable surface, $\#_{i=1}^{n} \mathbb{R}P_{i}^{2}$, that is embedded in 4-space is in $\{-2n, -2n + 2, \ldots, 2n - 2, 2n\}$. 

Remark. Only in the case that $E = 0$ does such a surface bound a Seifert solid in 4-space.
Whitney’s conjecture/Massey’s Theorem

The normal Euler class $E$ of a non-orientable surface, $\#_{i=1}^{n} \mathbb{RP}^2_i$, that is embedded in 4-space is in \{-2n, -2n + 2, \ldots, 2n - 2, 2n\}. Remark. Only in the case that $E = 0$ does such a surface bound a Seifert solid in 4-space.
• ¿Is this example the connected sum of the 3-twist spun trefoil and a standard projective plane?
Commentary

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- ¿Is it standard?
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• ¿If not, what invariants can be found?
• ¿Fold set?
Problems

$K$ — a classical knot
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$S(2n + 1) = T^{\pm(2n+1)}(K)$ — an odd twist spun of

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\(S(2n + 1) \# \mathbb{R}P^2\) — the connected sum of \(S\) with a standard \(\mathbb{R}P^2\) with normal Euler class \(\pm 2\).
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¿Is $S(2n + 1)$ standard?

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¿Is $S(2n + 1)$ standard?
¿Is $S(2n + 3)$ isotopic to $S(2n + 1)$ for all $n$?
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The general solution to the knotting problem will give information about “fake” $\pm \mathbb{CP}^2$s — spaces that are homeomorphic to, but not diffeomorphic to, $\pm \mathbb{CP}^2$.  


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The general solution to the knotting problem will
give information about “fake” $\pm \mathbb{CP}^2$s — spaces
that are homeomorphic to, but not diffeomorphic
to, $\pm \mathbb{CP}^2$. Clearly, the sign of the $\mathbb{CP}^2$ is related
to the normal Euler class.
Techniques

• Movie moves, chart moves, and decker sets
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- Cocycle invariants for quandles that have a good involution
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- Cocycle invariants for quandles that have a good involution
- Put in extra cusp pairs, via lips moves, to make the surface more flexible — to do so might allow some cancelation of triple points.
Thank you very much

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ありがとう
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