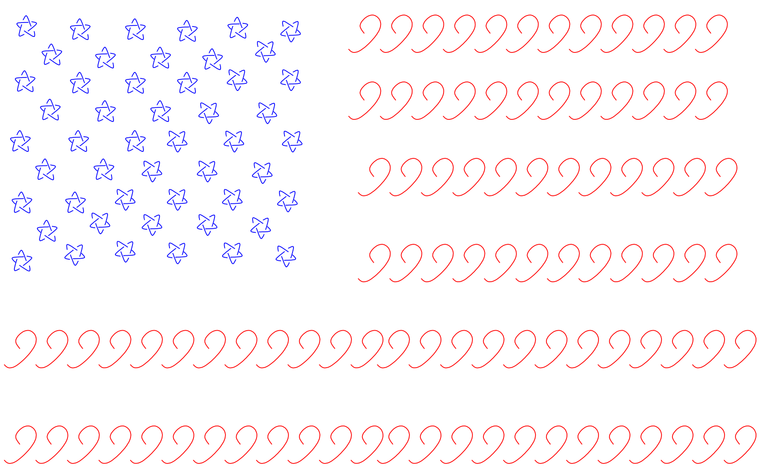
Infiltration of eloborate schemes

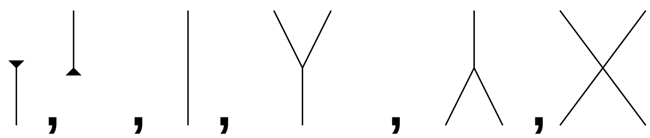
joint with/ Alissa Crans, Mohamed Elhamdadi

Pedro Lopes, and Masahico Saito

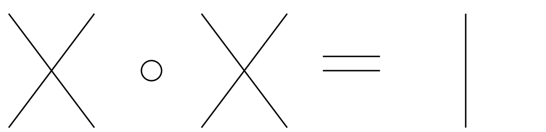
Knots in Washington:

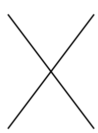


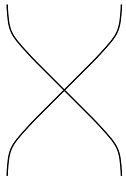
We consider a monoidal cat. in which objects correspond to non-negative integers, morphisms are generated by

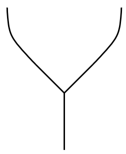


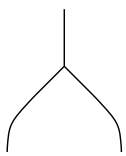
. Hom(A,B) has the structure of an Abelian group.

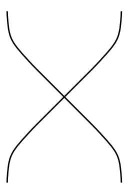
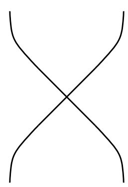
 **.**

And  **is natural with respect to all morphisms.**

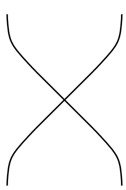
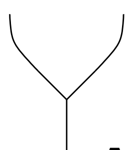
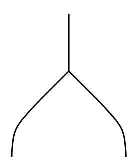
 : [2] $\longrightarrow$ [2]

 :[1] $\longrightarrow$ [2]

 : [2] $\longrightarrow$ [1]

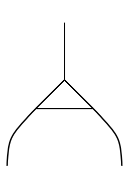
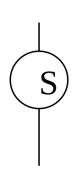
 =  1,1

Tensor powers,

 A,B ,  A ,  A,A

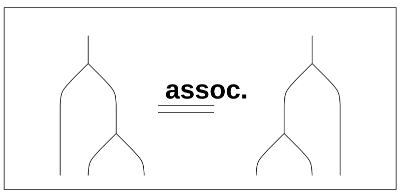
are easy to define inductively.

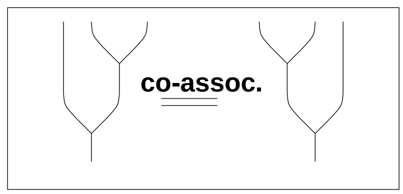
Other morphisms such as:

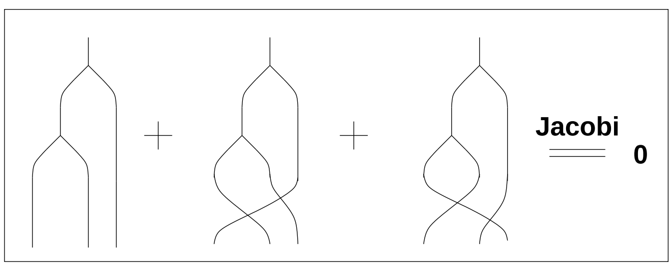
 

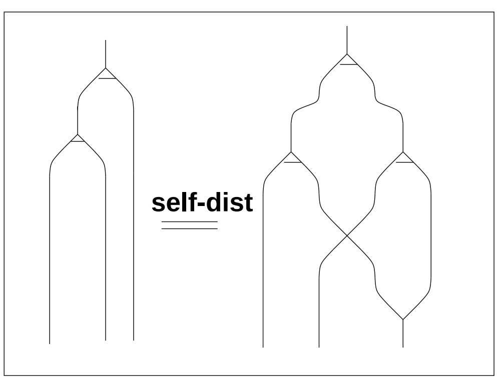
may be added to the cat.

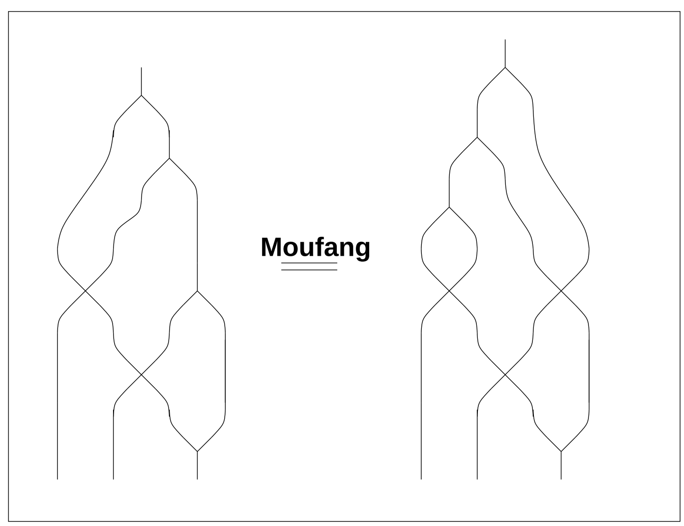
We use this category to model axioms such as associativity, co-assoc., Jacobi id, Moufang loops, and self-distributivity.

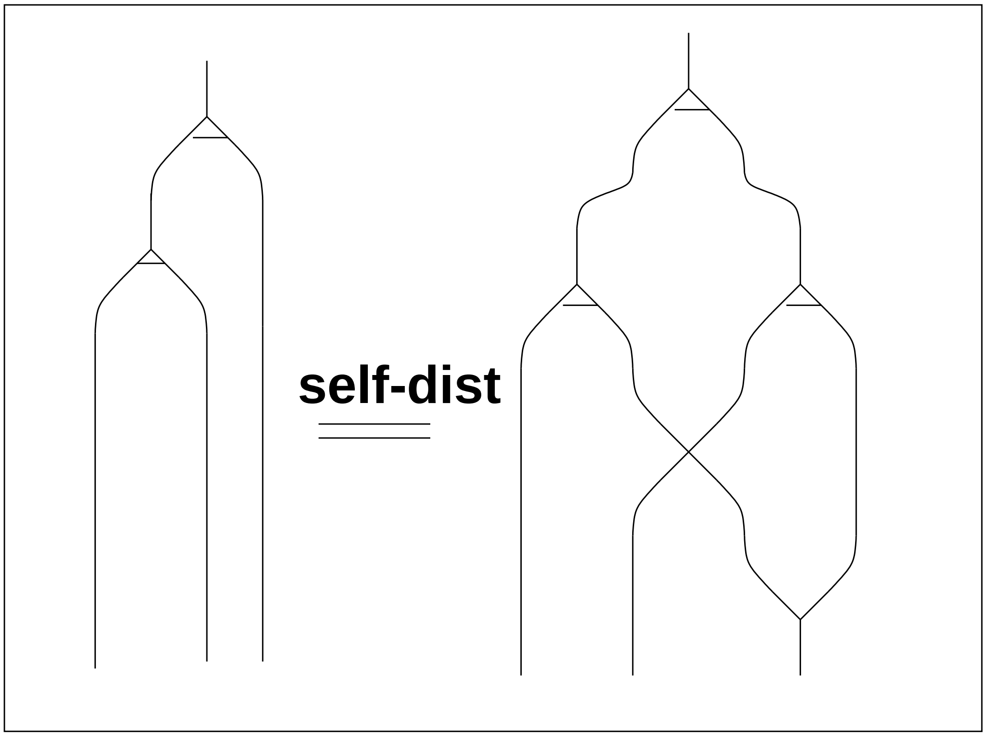




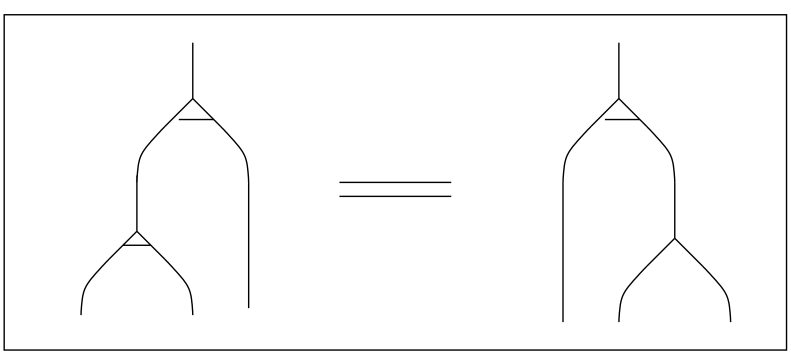




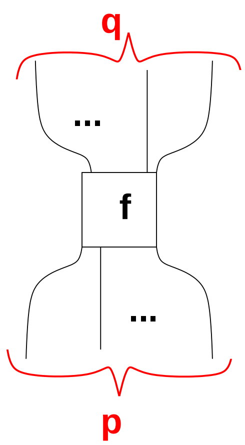




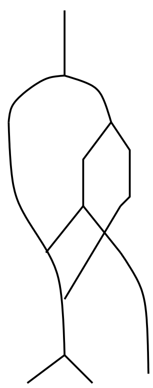
Here I have introduced another morphism in the category. The reason is that in quandles induced by conjugation, we also have the relation:



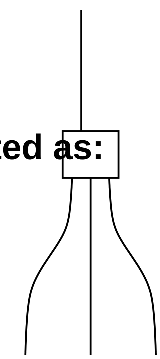
In general, we have morphisms, f, of the form



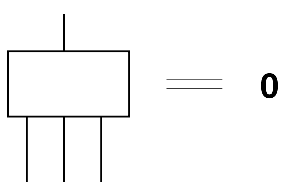
For example,



may be represented as:

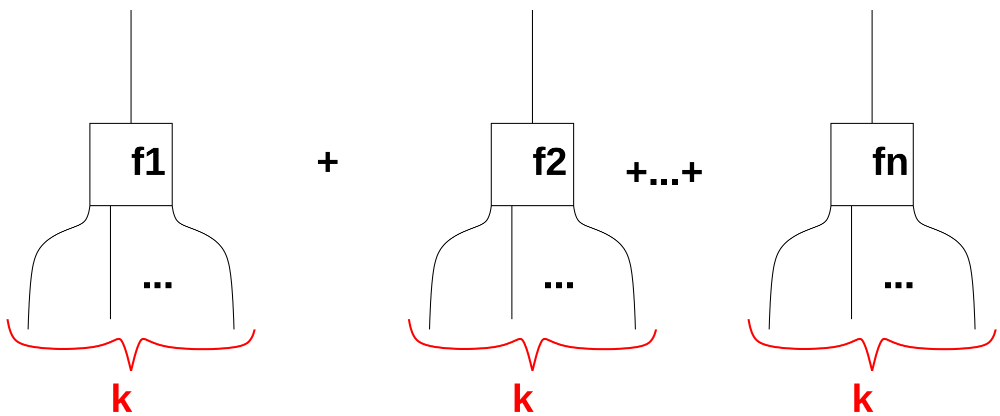


Or the Jacobi identity might be represented as



where the empty box represents the sum of the [[**]*] products.

An elaborate scheme of order (k,1) is a linear combination:

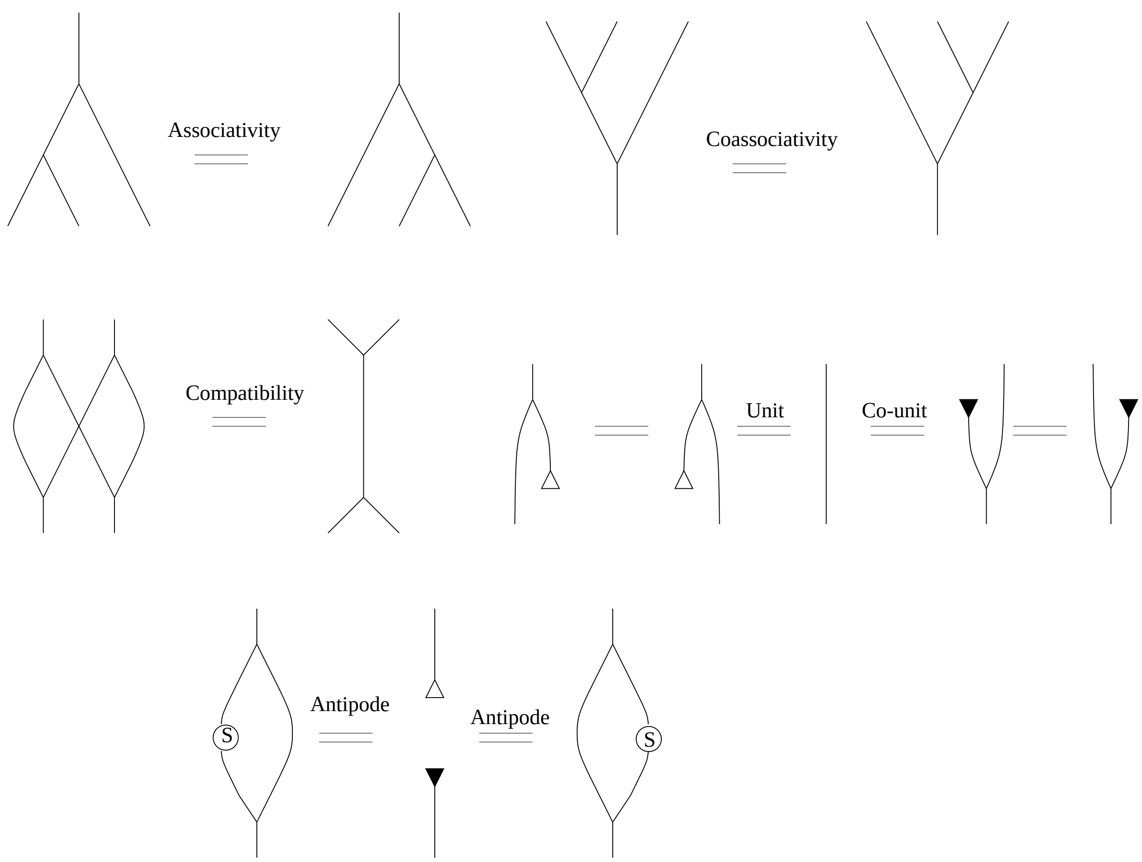


For example,

the Jacobi identity, associativity, coassociativity or the Moufang id. are examples in which an elaborate scheme is set to 0.

To model algebras, coalgs. etc, we take Cat/(eloboarte schemes)

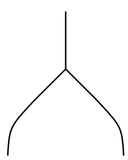
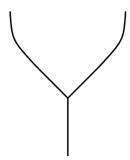
These are the axioms for a Hopf alg:



An infiltration of an elaborate scheme

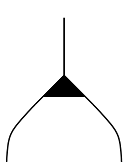
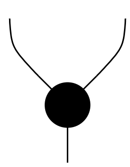
works as follows:

Exactly one factor of the form

 **or** 

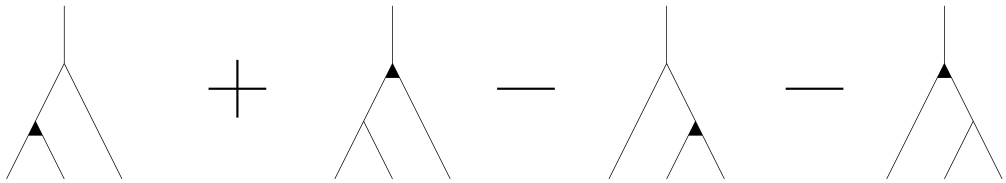
in each term of the elaborate scheme

is replaced with

 **or**  **resp.**

where the darkened symbols are fixed morphism.

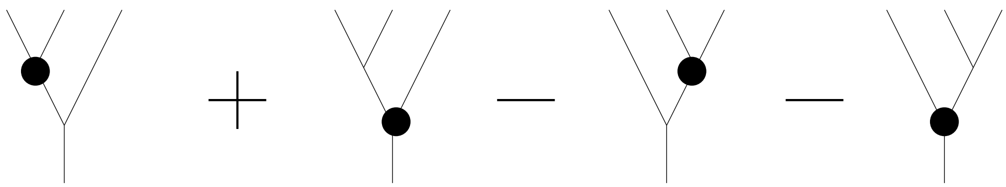
Here is an infiltration of assoc:



Thus an Infiltration correspond to a differential:

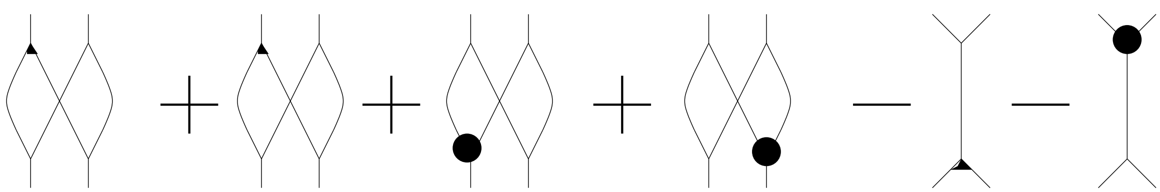
Let f(a,b)={a|b}

df(a,b,c)={a|b}c +{ab|c} - a{b|c} -{a|bc}



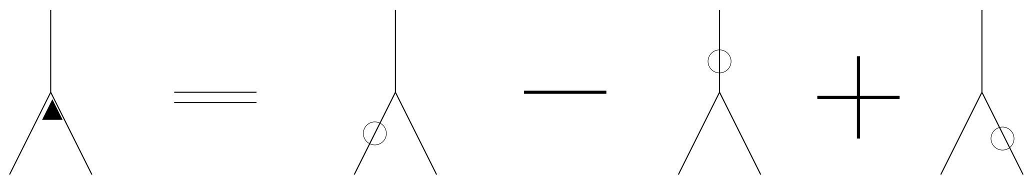
Comultiplication is more notationally complicated.

$\{a_1\}_1 \otimes \{a_1\}_2 \otimes a_2 + \{a_1 \otimes a_2\}_1 \otimes \{a\}_2 - a_1 \otimes \{a_2\}_1 \otimes \{a_2\}_2 - \{a\}_1 \otimes \{a_1 \otimes a_2\}_2$

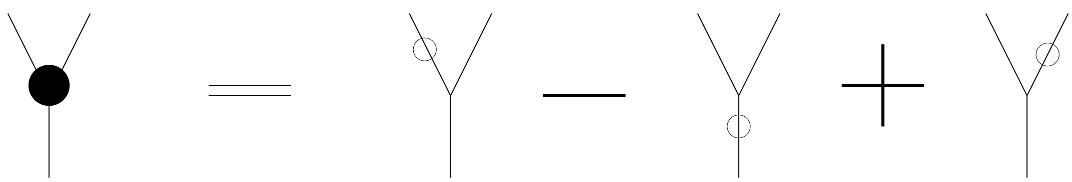


This expression coincides with the Hochschild differential of a pair of maps (f,g)

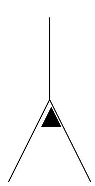
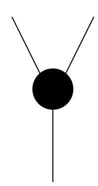
Infiltrate (ab):

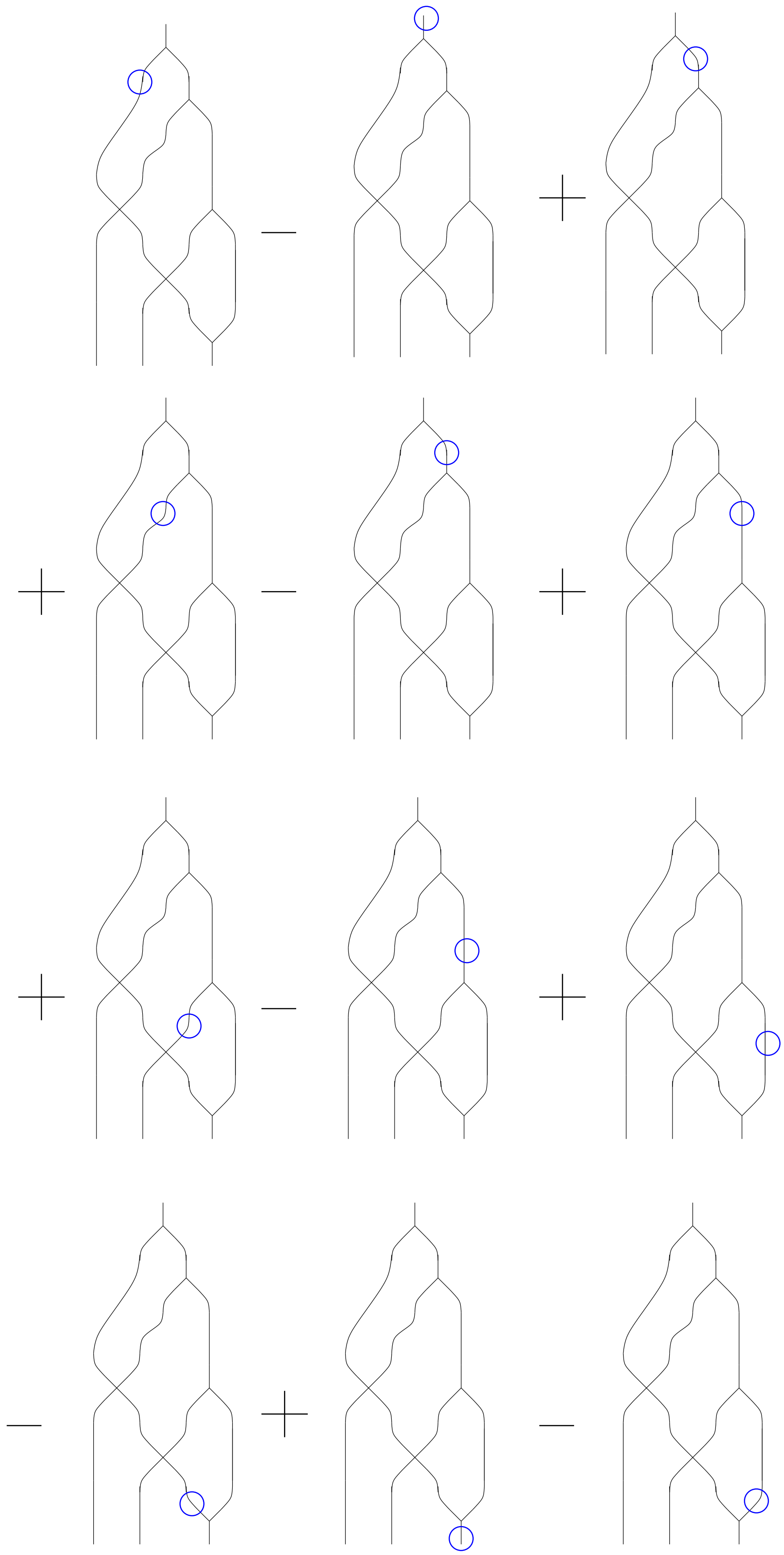


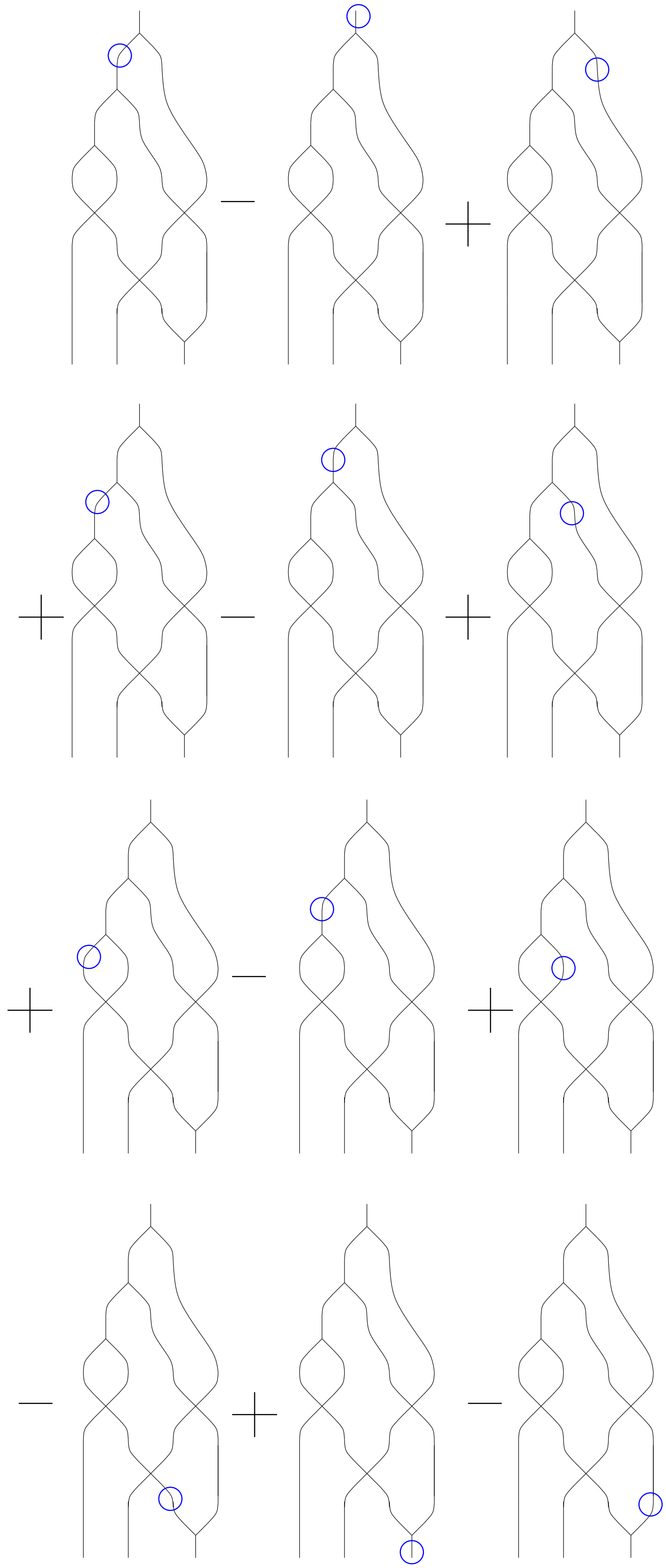
{a}b -{ab}+a{b}= d g(a,b) where g(x) is denoted by {x}.



OK NOW SET D(f,g)= f-g where

f=  **, g=** 





OK, what does this mean?

1. Infiltrations correspond to differentials in some coh. thy.

2. In the Gerstenhaber deformation thy, these differentials show up, regardless of the algebraic context.

3. Sabinin Algebras. (such as alg over loops).

4. The three dimensional thy, needs to be worked out in general: Identities among relations ...

5. Mohamed's talk will focus on the deformation of the adjoint.