

Describing the 2-twist-spun trefoil for humans and
robots

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Based on joint work e.g. with Saito and Kamada

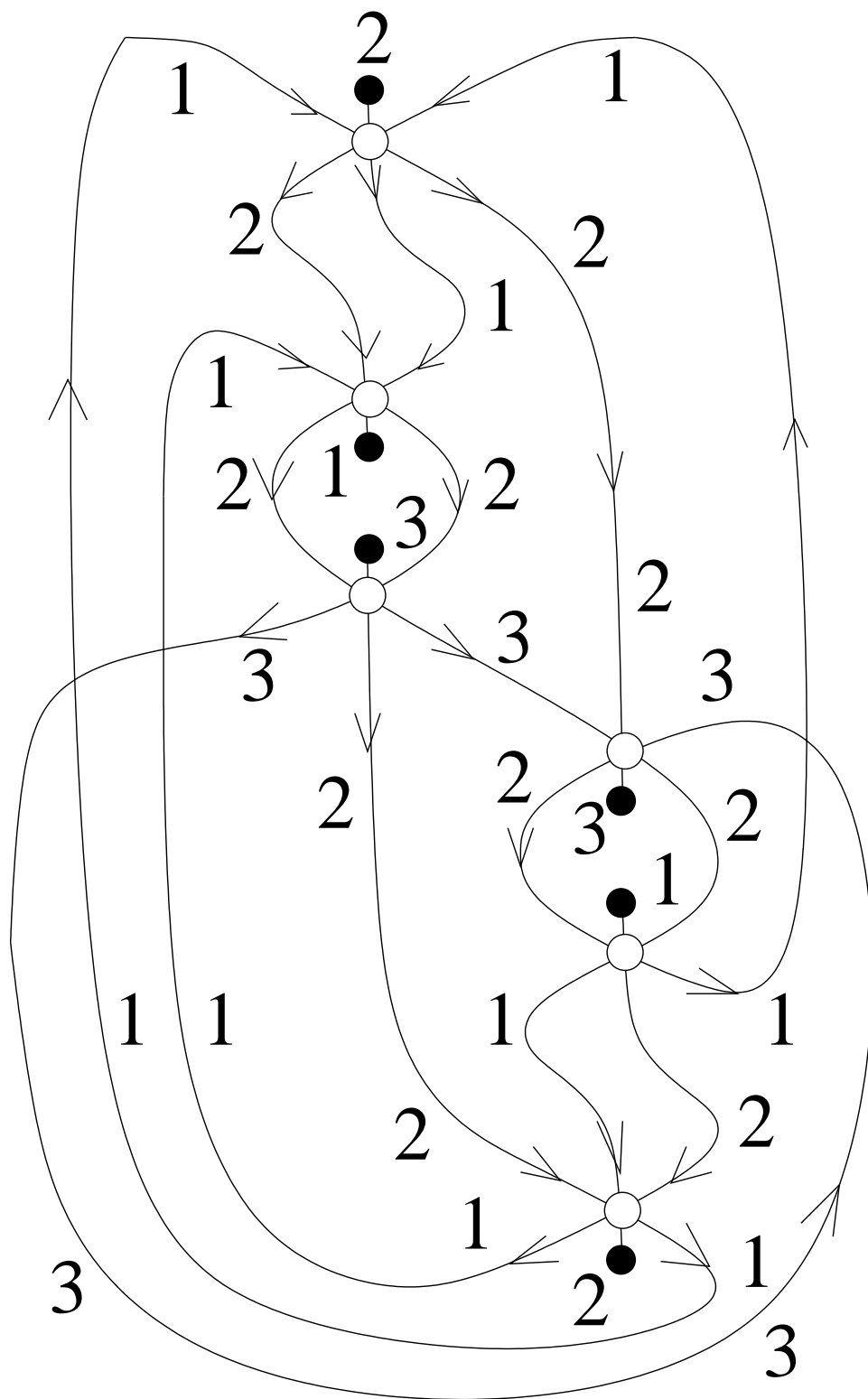
- Give descriptions of a non-trivial knotted sphere that are sufficient to compute KhoHo.
- Show how to attach a trivial 1-handle so torus remains knotted.
- Example 1st introduced by Fox:

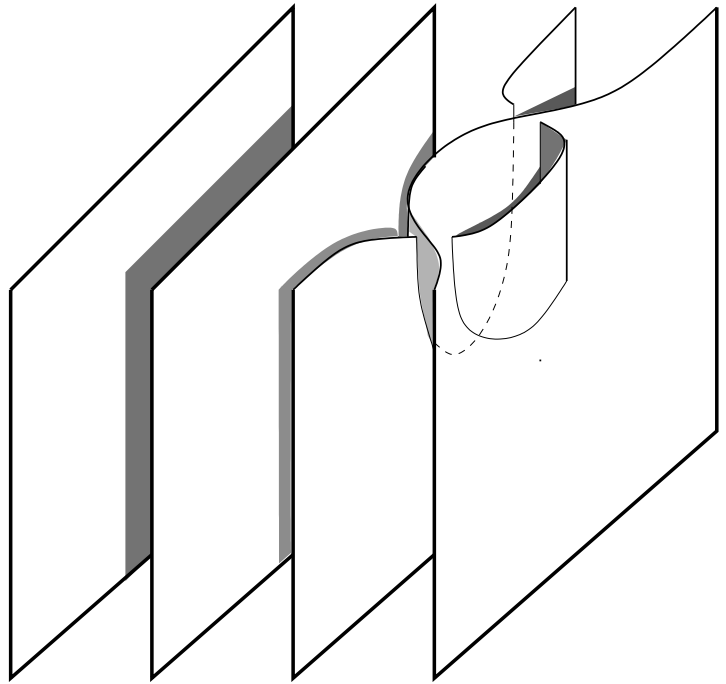
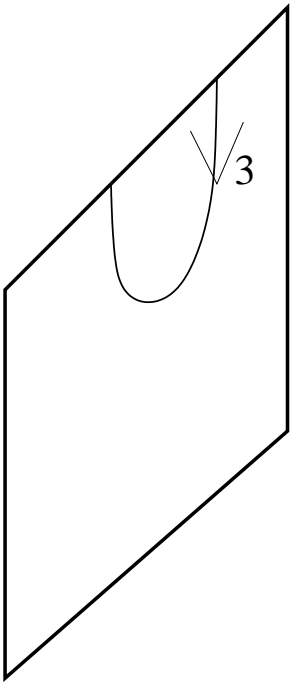
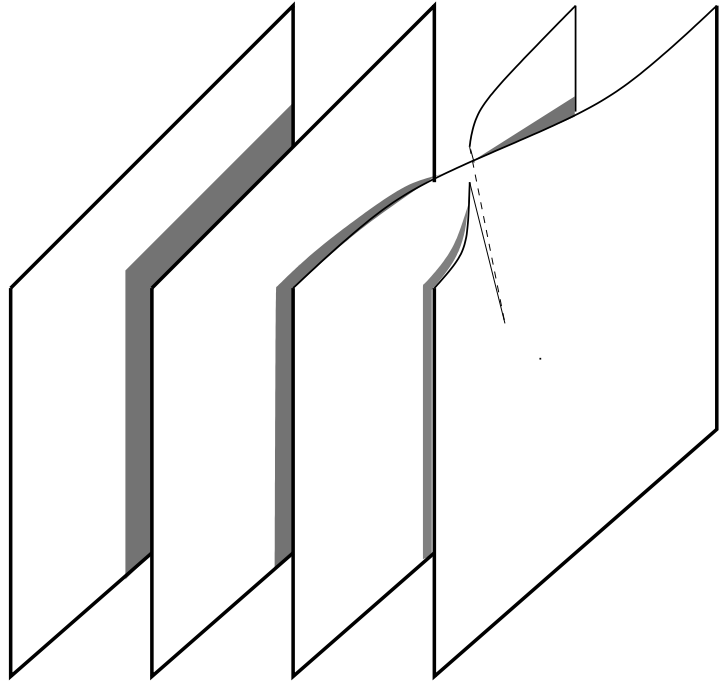
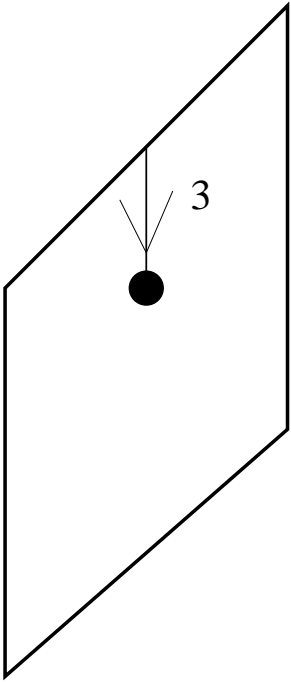
$$H_1(\tilde{X}) = \mathbf{Z}[t, t^{-1}] / (2t - 1, 2 - t)$$

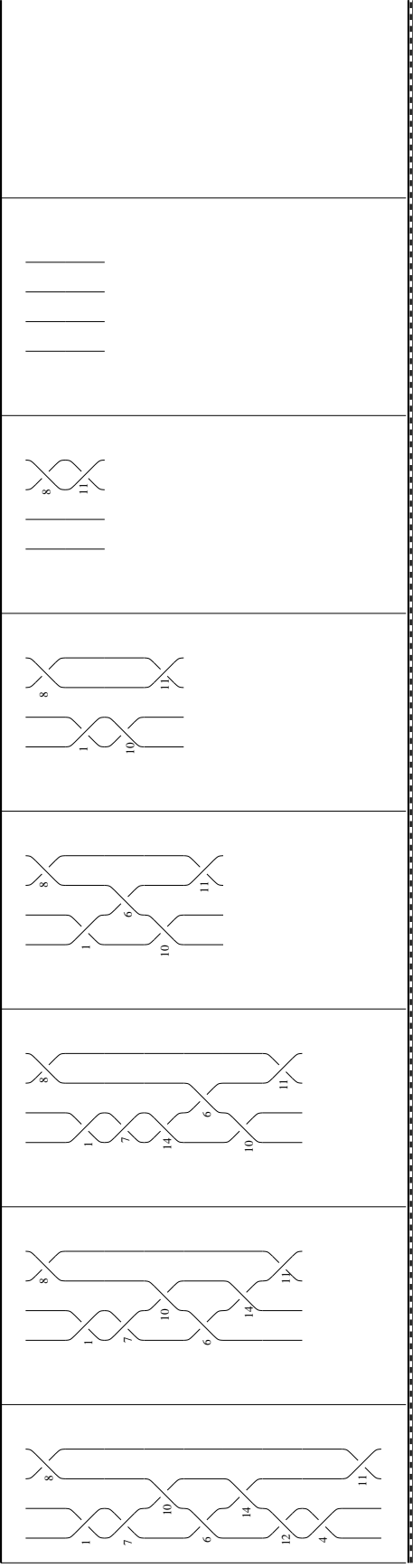
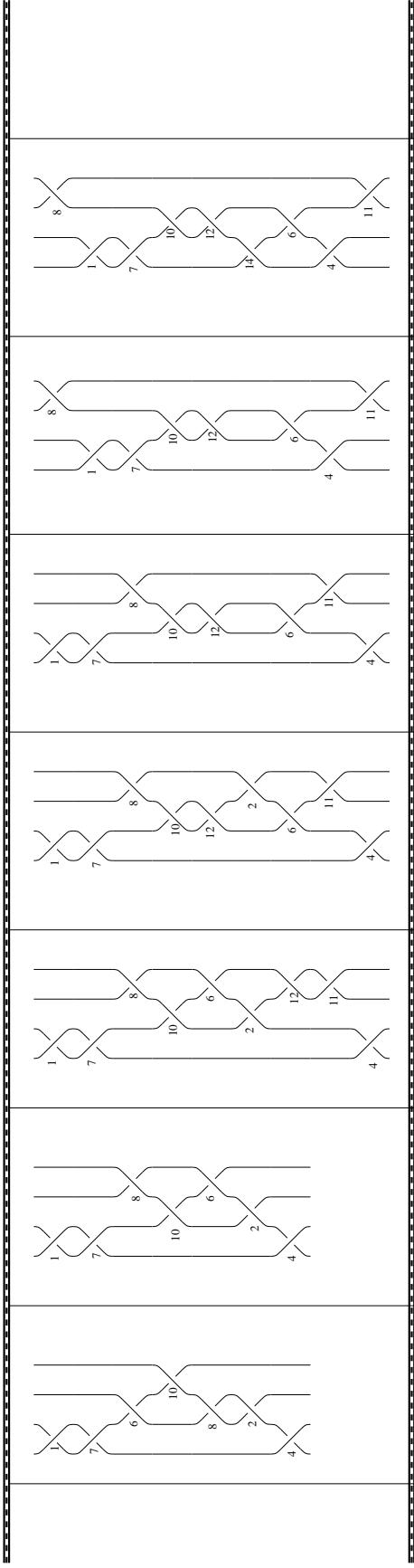
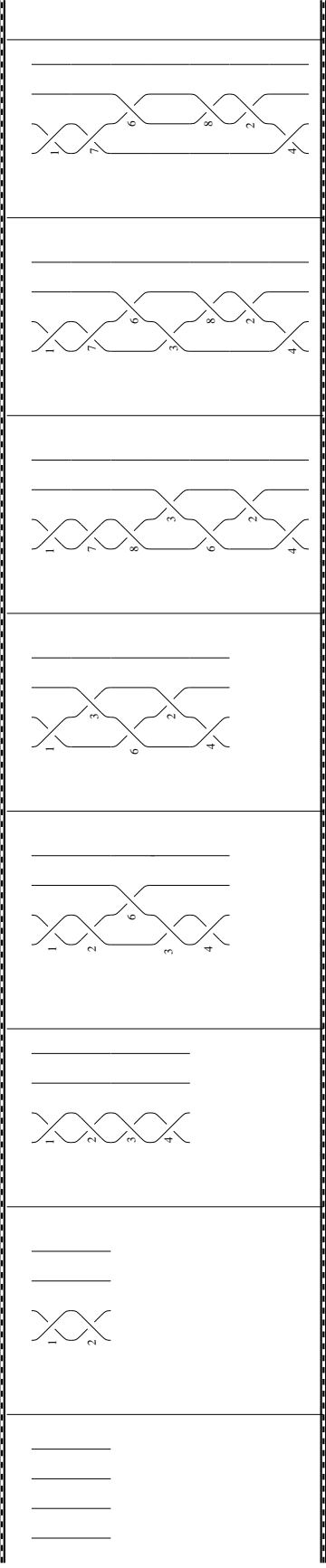
- Zeeman: $S^4 \setminus K$ is fibred with fibre:

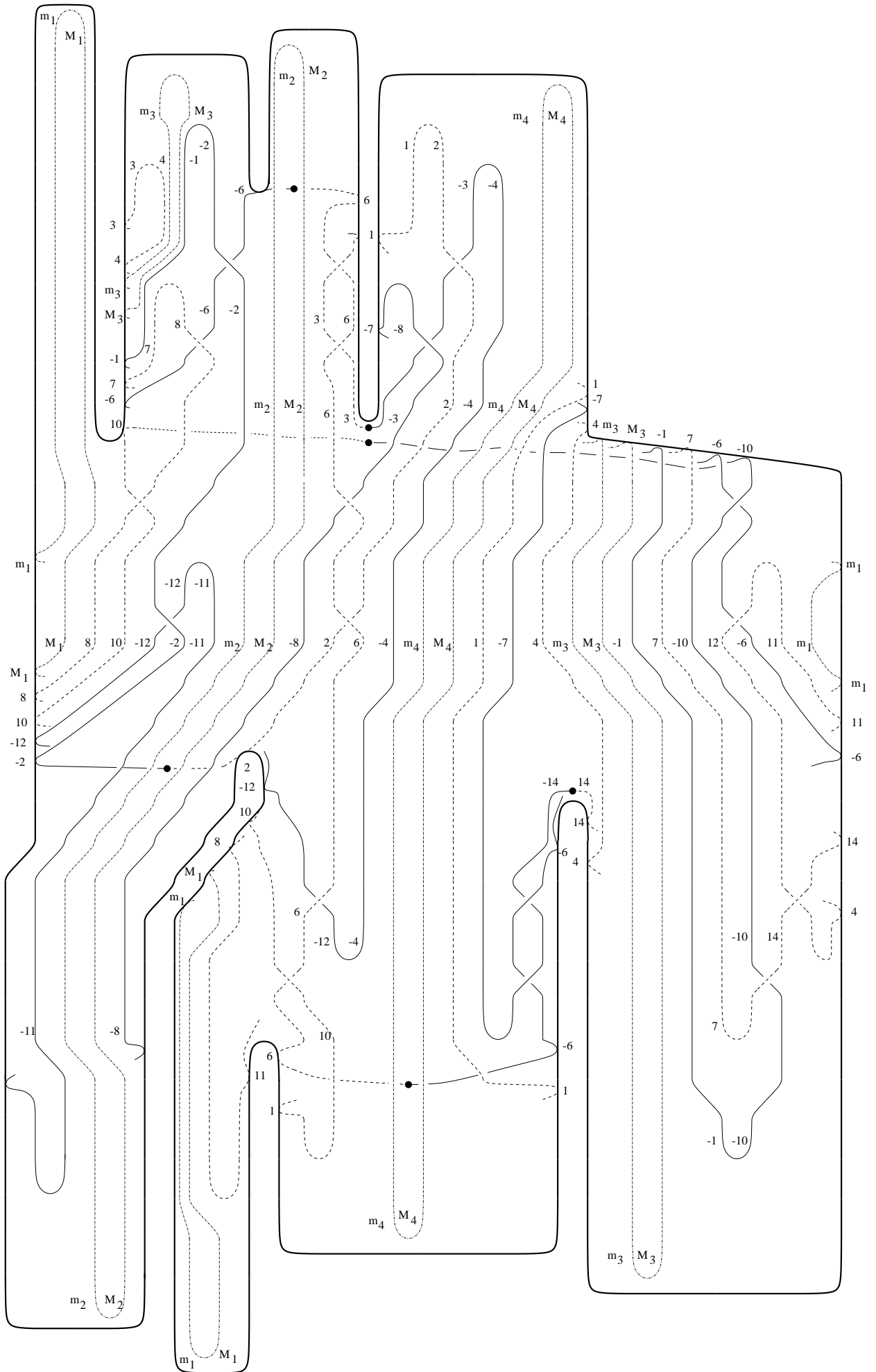
$L(3, 1) = 2$ -fold branched cover of S^3 branched along trefoil.

- Non-invertible
 - (Hillman ~1980 using Faber Levine)
 - (Ruberman using Casson-Gordon)
 - CJKLS using quandle coycle invariants
- Any projection has at least 4 triple points (Satoh Shima)
- Braid index is 4 (Kamada)









Briad Description for Robots

may require human intervention

{ {} ,

(* - II -> *)

{ sb[1] , s[1] } ,

(* = *)

{ sb[1] , s[1] , (*in*) } ,

(* - II -> *)

{ sb[1] , s[1] , s[1] , sb[1] } ,

(* = *)

{ sb[1] , s[1] , (*in*)s[1] , sb[1] } ,

(* - BranchBirth[2] -> *)

{ sb[1] , (s[1] , s[2] , s[1]) , sb[1] } ,

(* - III -> *)

{ sb[1] , (s[2] , s[1] , s[2]) , sb[1] } ,

(* = *)

{ sb[1] , (*in*)s[2] , s[1] , s[2] , sb[1] } ,

(* - II -> *)

{ sb[1] , sb[1] , (s[1] , s[2] , s[1]) , s[2] , sb[1] } ,

(*-III->*)

{sb[1], sb[1], s[2], (s[1]), s[2], s[2], sb[1]},

(*-BranchDeath[1]->*)

{sb[1], sb[1], s[2], (*in*)s[2], s[2], sb[1]},

(*-BranchBirth[3]->*)

{sb[1], sb[1], (s[2], s[3], s[2]), s[2], sb[1]},

(*-III->*)

{sb[1], sb[1], s[3], s[2], s[3], s[2], sb[1]},

(*=*)

{sb[1], sb[1], s[3], s[2], s[3], s[2], (*in*)sb[1]},

(*-II->*)

{sb[1], sb[1], s[3], s[2], (s[3], s[2], s[3]), sb[3], sb[1]},

(*-III->*)

{sb[1], sb[1], s[3], s[2], s[2], (s[3]), s[2], sb[3], sb[1]},

(*-BranchDeath[3]->*)

{sb[1], sb[1], s[3], s[2], s[2], s[2], (sb[3], sb[1])},

(*-X(1,3)->*)

{sb[1], (sb[1], s[3]), s[2], s[2], s[2], sb[1], sb[3]},

(*-X(1,3)->*)

{(sb[1], s[3]), sb[1], s[2], s[2], s[2], sb[1], sb[3]},

(*X(1,3)->*)

{s[3],sb[1],sb[1],s[2],s[2],(*in*),s[2],sb[1],sb[3]},

(*BranchBirth[1]->*)

{s[3],sb[1],sb[1],s[2],(s[2],s[1],s[2]),sb[1],sb[3]},

(*III->*)

{s[3],sb[1],sb[1],s[2],s[1],s[2],(s[1],sb[1]),sb[3]},

(*II->*)

{s[3],sb[1],sb[1],(s[2],s[1],s[2]),sb[3]},

(*III->*)

{s[3],sb[1],(sb[1],s[1]),s[2],s[1],sb[3]},

(*II->*)

{s[3],sb[1],(s[2]),s[1],sb[3]},

(*BranchDeath[2]->*)

{s[3],(sb[1],s[1],)sb[3]},

(*II->*)

{s[3],sb[3]},

(*II->*) {} }

Gauss Morse Description for Robots

may require human intervention

{ {}, (*=Birth=>*)

{mi[1], ma[1]},

(*=Birth=>*)

{{mi[1], ma[1]},

{mi[2], ma[2]}}},

(*=Birth=>*)

{{mi[1], ma[1]},

{mi[3], ma[3]},

{mi[2], ma[2]}}},

(*=Birth=>*)

{{mi[1], ma[1]},

{mi[3], ma[3] (*in*)},

{mi[2], ma[2]},

{(*in*)mi[4], ma[4]}}},

(*=II=>*)

{{mi[1], ma[1]},

{(*in*)mi[3], ma[3], -1, -2},

{mi[2], ma[2]}, {1, 2, (*in*)mi[4], ma[4]}}},

(*=II=>*)

{{mi[1], ma[1]},

{3,4,mi[3],ma[3],-1,-2 (*=>*)},

{(*<=*)mi[2],ma[2]},

{1,2,-3,-4,mi[4],ma[4]}}},

(*=Branch/Saddle=>*)

{{mi[1], ma[1]},

{(6,3),4,mi[3],ma[3],-1,(-2,-6),mi[2],ma[2]},

{1,(2,-3),-4,mi[4],ma[4]}}},

(*=Type III=>*)

{{mi[1], ma[1]},

{3,6,4,mi[3],ma[3],-1,-6,-2,mi[2],ma[2]},

{1,-3,2,-4,mi[4],ma[4]}}},

(*=*)

{{mi[1], ma[1]},

{4,mi[3],ma[3],-1,(*in*)-6,-2,mi[2],ma[2],3,6},

{1,(*in*)-3,2,-4,mi[4],ma[4]}}},

(* =Type II =*)

{{mi[1], ma[1]},

{4,mi[3],ma[3],-1,7,(8,-6),-2,mi[2],ma[2],(3,6)},

{1,-7,(-8,-3),2,-4,mi[4],ma[4]}}},

(*Type III=>*)

{{mi[1], ma[1]},

{4,mi[3],ma[3],-1,7,-6,8,-2,mi[2],ma[2],6,3},

{1,-7,-3,-8,2,-4,mi[4],ma[4]}}},

(*=*)

{{mi[1], ma[1]},

{4,mi[3],ma[3],-1,7,-6,8,-2,mi[2],ma[2],6,(*=>*)3},

{-3,(*<=*)-8,2,-4,mi[4],ma[4],1,-7}}},

(*Branch/Saddle=>*)

{{mi[1], ma[1]},

{4,mi[3],ma[3],-1,7,-6,8,-2,mi[2],

ma[2],6,-8,2,-4,mi[4],ma[4],1,-7}}},

(*=*)

{{mi[1], ma[1](*=>*)},

{8,-2,mi[2],ma[2],6,-8,2,-4,mi[4],

ma[4],1,-7,4,mi[3],ma[3],-1,7,-6(*=>*)}}},

(*Branch/Saddle=>*)

{{mi[1], ma[1],(10,8),-2,mi[2],

ma[2],(6,-8),2,-4,mi[4],

ma[4],1,-7,4,mi[3],ma[3],-1,7,(-6,-10)}}},

(*=Type III=>*)

{{mi[1],ma[1],8,10,-2,(*in*)mi[2],
ma[2],-8,6,2,-4,mi[4],
ma[4],1,-7,4,mi[3],ma[3],-1,7,-10,-6(*in*)}},

(*=Type II=>*)

{{ mi[1],ma[1],8,10,(-2,-12),-11,mi[2],
ma[2],-8,(6,2),-4,mi[4],
ma[4],1,-7,4,mi[3],ma[3],-1,7,-10,(-6,12),11}},

(*=Type III=>*)

{{ mi[1],ma[1],8,10,-12,-2,-11,mi[2],
ma[2],-8,2,6,-4,mi[4],
ma[4],1,-7,4,mi[3],ma[3],-1,7,-10,12,-6,11}},

(***)

{{-2,(*<=*)-11,mi[2],
ma[2],-8,2,(*<=*)6,-4,mi[4],
ma[4],1,-7,4,mi[3],ma[3],-1,7,-10,12,-6,11,mi[1],
ma[1],8,10,-12}},

(*=Branch/Saddle=>*)

{{-11,mi[2],ma[2],-8},
{6,-4,mi[4],ma[4],1,-7,4,mi[3],
ma[3],-1,7,-10,12,-6,11,mi[1],ma[1],8,10,-12}},

(* = *)

{{-11, mi [2], ma [2], -8},

{ -6, 11, mi [1], ma [1], 8, 10, -12, 6, -4, mi [4],
ma [4], 1, -7, (*<=>*)4, mi [3],

ma [3], -1, 7, -10, 12}}},

(* = Branch/Saddle => *)

{{-11, mi [2], ma [2], -8},

{-6, 11, mi [1], ma [1], 8, 10, -12, 6, -4, mi [4],
ma [4], 1, -7, -14},

{14, 4, mi [3], ma [3], -1, 7, -10, 12}}},

(* = *)

{{-11, mi [2], ma [2], -8},

{11, mi [1], ma [1], 8, 10, (-12, 6), -4, mi [4],
ma [4], 1, -7, (-14, -6)},

{4, mi [3], ma [3], -1, 7, -10, (12, 14)}}},

(* = Type III => *)

{{-11, mi [2], ma [2], -8},

{11, mi [1], ma [1], 8, 10, 6, -12, -4, mi [4],
ma [4], 1, -7, -6, -14},

{4, mi [3], ma [3], -1, 7, -10, 14, 12}}},

(* = *)

{{mi [2], ma [2], -8, -11},

{11, mi [1], ma [1], 8, 10, 6, (-12, -4), mi [4],

ma [4], 1, -7, -6, -14},

{mi [3], ma [3], -1, 7, -10, 14, (12, 4)}}},

(* = Type II => *)

{{mi [2], ma [2], -8, -11},

{11, mi [1], ma [1], 8, (10, 6), mi [4],

ma [4], 1, -7, (-6, -14)},

{mi [3], ma [3], -1, 7, (-10, 14)}}},

(* = Type III => *)

{{mi [2], ma [2], -8, -11},

{11, mi [1], ma [1], 8, 6, 10, mi [4],

ma [4], 1, (-7, -14), -6},

{mi [3], ma [3], -1, (7, 14), -10}}},

(* = Type II => *)

{{mi [2], ma [2], -8, -11},

{11, mi [1], ma [1], 8, (* => *) 6, 10, mi [4],

ma [4], 1, -6 (* <= *)},

{mi [3], ma [3], -1, -10}}},

(*Branch/Saddle=>*)

{{mi[2],ma[2],-8,-11},

{11, mi[1],ma[1],8},{10,mi[4],ma[4],1},

{mi[3],ma[3],-1,-10}}},

(*=*)

{{mi[2],ma[2],-8,-11},

{11, mi[1],ma[1],8},

{1,10,mi[4],ma[4]},

{mi[3],ma[3],-1,-10}}},

(*=*)

{{mi[2],ma[2],-8,-11},

{11, mi[1],ma[1],8},

{1,10,mi[4],ma[4]},

{mi[3],ma[3],-1,-10}}},

(*TypeII=>*)

{{mi[2],ma[2],-8,-11},

{11, mi[1],ma[1],8},

{mi[4],ma[4]},

{mi[3],ma[3]}}

(* = *)

{{mi [2], ma [2], -8, -11},

{8, 11, mi [1], ma [1]},

{mi [4], ma [4]},

{mi [3], ma [3]}}

(* = TypeII => *)

{{mi [2], ma [2]},

{mi [1], ma [1]},

{mi [4], ma [4]},

{mi [3], ma [3]}}

(* = Death => *)

{{mi [2], ma [2]},

{mi [1], ma [1]}

{mi [3], ma [3]}}

(* = Death => *)

{{mi [2], ma [2]},

{mi [1], ma [1]}}

(* = Death => *)

{{ mi [1], ma [1]}}

(* = Death => *)

{}}

