Categorical Quandles and Knots

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Joint work with:

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Plan

1. State Main Results
2. Motivation
3. Categories in Groups and Quandles
4. The Functors Conj and Core
5. Example $TSU(2)$
6. Non-group examples
7. local arrow systems
8. The fundamental 2-quandle

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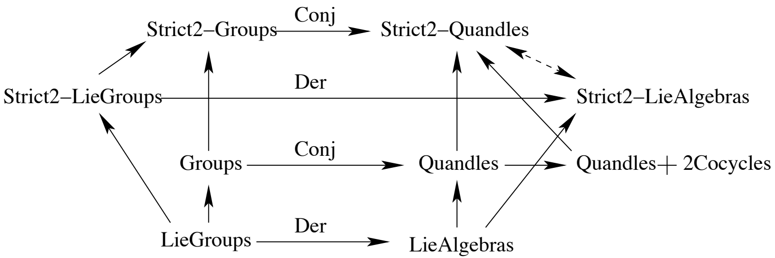
& some more stuff.

Motivations



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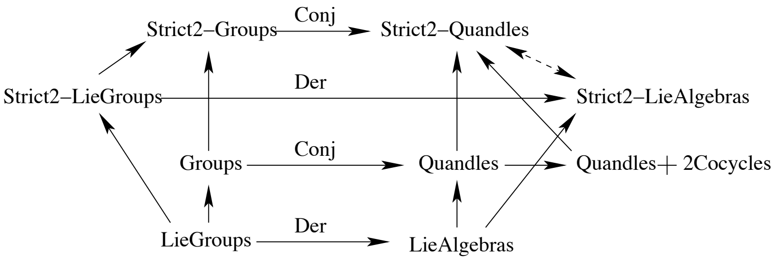

Relations via 3-cocycles ?!?!?!

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III. $\forall x, y, z \in X \quad (x \triangleleft y) \triangleleft z = (x \triangleleft z) \triangleleft (y \triangleleft z)$.

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Category in $\mathcal{C}$

Here $\mathcal{C}$ is either the catgory of groups or the category of quandles. If A is an object in $\mathcal{C}$, then its underlying set is denoted by $|A|$. A *category in* $\mathcal{C}$ is constructed:

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3. $(f_3 \circ f_2) \circ f_1 = f_3 \circ (f_2 \circ f_1)$;
4. $(i(x) \circ f_1) = f_1$, and $(f_2 \circ i(x)) = f_2$

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Lemma

If $(O, M, s, t, i, \circ)_{\mathcal{C}_0}$ *is a category in* $\mathcal{C}_0$*, then* $(\hat{O}, \hat{M}, \hat{s}, \hat{t}, \hat{i}, \hat{\circ})_{\mathcal{C}_1}$ *is a category in* $\mathcal{C}_1$*.*

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- target map $t(h, g) = \tau(h) g$,
- the id. $i(g) = (1, g)$,
- comp. $((h_2, \tau(g_1) h_1) \circ (h_1, g_1) = (h_2 h_1, g_1)$.

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$$(h_1, g_1) \triangleleft (h_2, g_2) = (\alpha(g_2^{-1}, h_2^{-1} h_1 \alpha(g_1, h_2)), g_2^{-1} g_1 g_2)$$

$$= (\alpha(g_2^{-1}, h_2^{-1}) \alpha(g_2^{-1}, h_1) \alpha(g_2^{-1}, \alpha(g_1, h_2)), g_2^{-1} g_1 g_2)$$

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So in the corresponding category, $M = TSU(2)$.

Another gratuitous formula

$$(a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}, \cos(s)\hat{\jmath} + \sin(s)\hat{k})$$

$$\triangleleft(a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}, \cos(t)\hat{\jmath} + \sin(t)\hat{k})$$

$$= ((2a_2 - a_1)\,\hat{\imath}$$

$$+ [b_1 \cos(2t)$$

$$+ 2\sin(s - 2t)(c_2\cos(s) - b_2\sin(s)) + c_1\sin(2t)]\,\hat{\jmath}$$

$$+ [\sin(2t)(b_1 + b_2\cos(2s) - b_2 + c_2\sin(2s))$$

$$+ \cos(2t)(-b_2\sin(2s) - c_1 + c_2\cos(2s) + c_2)]\,\hat{k},$$

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The point is that the representation of a 2-bridge knot will land on the (j, k)-circle in $SU(2)$. The formula gives the quandle action for this sub-quandle of $TSU(2)$.

Example, the right-handed trefoil

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- center arc next page

$$
\left(a_2\hat{\imath} + \left(\frac{-b_2 + \sqrt{3}c_2}{4} + \frac{\sqrt{3}}{2}(2c_2 - c_1) \right) \hat{\jmath} - \left(\frac{1}{2}(2c_2 - c_1) + \frac{\sqrt{3}}{4}(-b_2 + \sqrt{3}c_2) \right) \hat{k}, \frac{-1}{2}\hat{\jmath} - \frac{\sqrt{3}}{2}\hat{k} \right).
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So the space of quandle representations for the trefoil into $TSU(2)$ has 4 free parameters.

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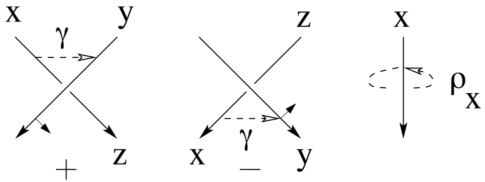
This will be called an *Alexander 2-quandle*.

Local Arrow Systems



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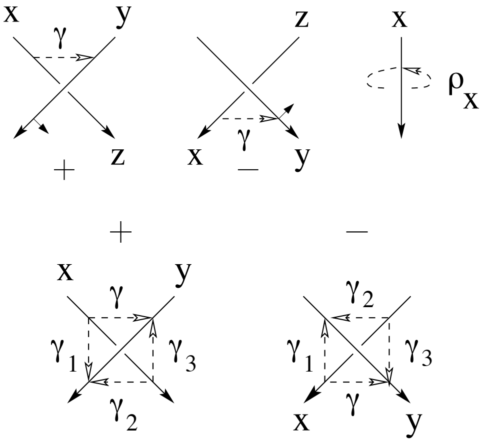

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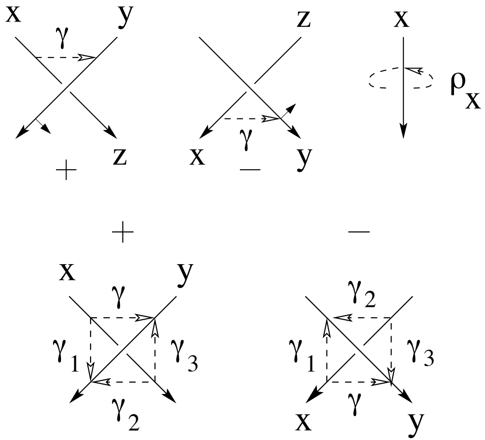

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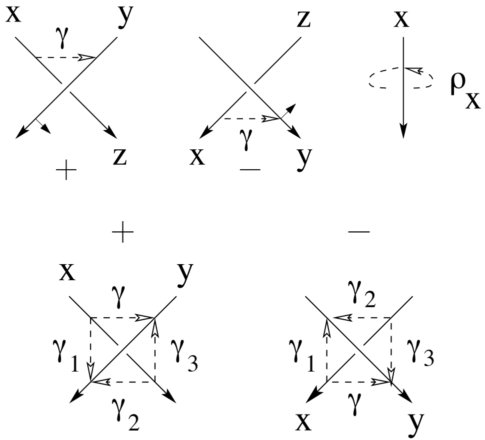

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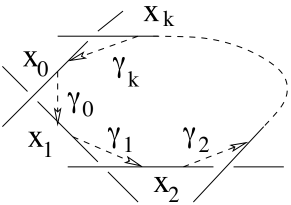

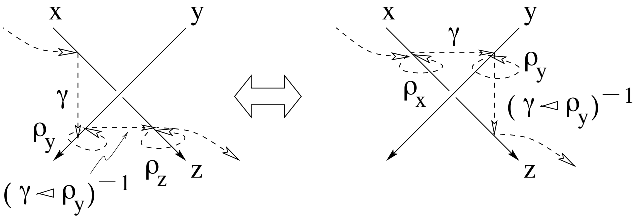

$$(\rho_z)\circ(\gamma\triangleleft\rho_y)^{-1}\circ(\rho_y)\circ(\gamma) = (\gamma\triangleleft\rho_y)^{-1}\circ(\rho_y)\circ(\gamma)\circ(\rho_x)$$

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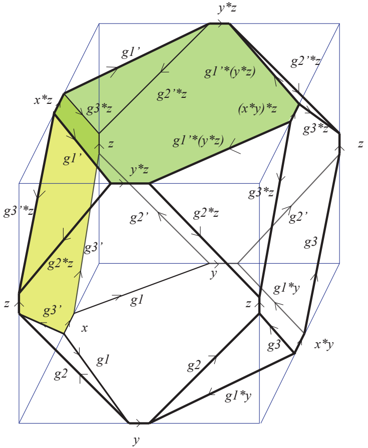

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