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Universal deformation rings of modules over Frobenius algebras.

2010 Mathematics Subject Classification: Primary 16G10; Secondary 16G20, 20C20

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This paper, portions of which are taken from the PhD thesis of the second author under the supervision of the first one, consists of two parts and two appendices.

The main result of the first part states that every finitely generated module over a finite dimensional self-injective algebra, whose stable endomorphism ring is isomorphic to the ground field, has a universal deformation ring. Moreover, if the algebra is Frobenius, then the universal deformation ring is invariant under taking syzygies. This generalizes results for group algebras of finite groups obtained by the first author and Chinburg [Math. Ann. **318** (2000), no. 4, 805–836; MR1802512 (2001m:20013)].

In the second part the authors determine the universal deformation rings for those finitely generated modules over a certain algebra $\Lambda_0$ of dihedral type over an algebraically closed field $k$ that have stable endomorphism rings isomorphic to $k$. Algebras of dihedral type were introduced by Erdmann in her classification of tame blocks of finite groups up to Morita equivalence. Note that $\Lambda_0$ is a Frobenius algebra that is not Morita equivalent to the block of a group algebra. Indeed, there are important differences between $\Lambda_0$ and blocks of group algebras with dihedral defect groups having precisely three isomorphism classes of simple modules, which were studied by the first author in [Trans. Amer. Math. Soc. **354** (2002), no. 10, 3893–3906; MR1926858 (2004a:20014)] and [Trans. Amer. Math. Soc. **361** (2009), no. 7, 3661–3705; MR2491895 (2010b:20015)]. Namely, the stable Auslander-Reiten quivers of these blocks have only finitely many components containing modules with stable endomorphism rings isomorphic to $k$ and none of the 1-tubes contain such modules, whereas the stable Auslander-Reiten quiver of $\Lambda_0$ has infinitely many components (of type $\mathbb{Z}A_\infty$) and infinitely many 1-tubes containing modules with stable endomorphism rings isomorphic to $k$.

In the first appendix the authors give explicit descriptions of those string and band modules for $\Lambda_0$ that have stable endomorphism rings isomorphic to $k$. The second appendix reviews a combinatorial description of the indecomposable $\Lambda_0$-modules (string and band modules), the homomorphisms between them, and a description of the components of the stable Auslander-Reiten quiver of $\Lambda_0$, which are needed in the second part of the paper.