Twenty-five years ago, L. J. Santharoubane [Proc. Amer. Math. Soc. 87 (1983), no. 1, 23–28; MR0677223 (84b:17010)] computed the cohomology of the $(2m + 1)$-dimensional Heisenberg Lie algebra $h_m$ with coefficients in any field of characteristic zero. Recently, E. Sköldberg [Math. Proc. R. Ir. Acad. 105A (2005), no. 2, 47–49; MR2171225 (2006e:16013)] determined the Poincaré polynomial of the homology of $h_m$ over fields of characteristic two by using algebraic discrete Morse theory. (Since the homology and the cohomology spaces of $h_m$ with coefficients in the ground field are isomorphic in any degree, Sköldberg’s result implies the analogue of Santharoubane’s result in characteristic two.)

The aim of the paper under review is to give an explicit formula for the Betti numbers of $h_m$ for any field of prime characteristic $p \geq 2$. (There is a typo in the formulation of the main result and in Corollary 1: “for all $i \leq m$” should be replaced by “for all $n \leq m$”.) Similarly to Santharoubane who uses induction on $m$, the author’s proceed in addition by induction on the degree of the relevant cohomology space. Since Santharoubane gives explicit cocycles and coboundaries in every degree in terms of the dual basis of $h_m$, it would be interesting to find a description of the additional cohomology classes in the modular case. As an application the authors show that the Betti numbers of any Heisenberg algebra in characteristic two are strictly unimodal whereas they are not unimodal in characteristic different from two (which was well-known in characteristic zero as a consequence of Santharoubane’s result.)