Yoneda defined extension groups $\text{Ext}^d_A(M, N)$ for objects $M, N$ of an abelian category $A$, and any non-negative integer $d$, as the set of all equivalence classes of exact sequences of length $d + 2$ starting in $N$ and terminating in $M$. For a full abelian subcategory $B$ of $A$ with exact inclusion functor $B \to A$ there are canonical maps $\text{Ext}^d_B(M, N) \to \text{Ext}^d_A(M, N)$ which in general are neither injective nor surjective. Then $B$ is called extension full in $A$ if these canonical maps are isomorphisms for all objects $M, N$ of $B$ and any non-negative integer $d$. A Guichardet category is an abelian category $A$ of finite global dimension such that every initial segment in $A$ is extension full in $A$. These properties are motivated by a result of Cline, Parshall, and Scott [J. Reine Angew. Math. 391 (1988), 85–99; MR0961165] stating that the Serre subcategory associated to a coideal of the partially ordered set indexing the simple objects of some highest weight category $C$ is extension full in $C$.

In the first part of the paper under review the authors prove several criteria for extension fullness in a general abstract setting. This is then applied to show that for a finite-dimensional complex semisimple Lie algebra $g$ the BGG category $O$ is extension full in the category of all weight modules and the thick BGG category $O^\infty$ is extension full in the category of all $g$-modules. The authors also prove a similar result for basic classical Lie superalgebras. Then it is shown that regular blocks of the (thick) BGG category are Guichardet but already for $\mathfrak{sl}_3$ there is a singular block of category $O$ that is not Guichardet. In particular, the principal block of the (thick) BGG category is Guichardet. The analogous statement for the principal block of the category of Harish-Chandra bimodules is the so-called weak Alexandru conjecture which is confirmed in the thick case as then these blocks are equivalent to certain regular blocks of the (thick) BGG category. On the other hand, the authors show that not all regular blocks of a certain category of Harish-Chandra bimodules are extension full in the category of all $g$-bimodules.