Du, Jie; Fu, Qiang:  
A modified BLM approach to quantum affine $\mathfrak{gl}_n$.  

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A. A. Beilinson, G. Lusztig, and R. MacPherson [Duke Math. J. **61** (1990), no. 2, 655–677; MR1074310 (91m:17012)] constructed the quantized universal enveloping algebra $U(n)$ of $\mathfrak{gl}_n$ from the quantum Schur algebra $S(n, r)$ and its geometric setting by singling out a certain spanning set and giving explicit multiplication formulas between generators and the elements of the spanning set. A certain triangular relation makes it possible to define an infinite-dimensional non-unital limit algebra $K(n)$. In the completion $\hat{K}(n)$ of $K(n)$ the lifted spanning set from the one in the quantum Schur algebra is linearly independent, the so-called BLM-basis of $U(n)$, and spans a subalgebra which is isomorphic to $U(n)$.  

In the paper under review the authors develop the affine analogue of the Beilinson-Lusztig-MacPherson approach. Most of the BLM machinery (multiplication formulas and spanning set) generalizes to the affine case and yields an explicit algebra homomorphism from the quantized universal enveloping algebra $U_\Delta(n)$ of affine $\mathfrak{gl}_n$ to the quantum affine Schur algebras $S_\Delta(n, r)$. But the generalization of the triangular relation cannot be used to define the affine analogue of the limit algebra $K(n)$. Instead the authors replace $K(n)$ by the direct sum $\mathcal{K}_\Delta(n)$ of all the $S_\Delta(n, r)$ for any non-negative integers $r$. Then Ringel-Hall theory enables them to establish a certain triangular relation in $S_\Delta(n, r)$ and in the completion $\hat{\mathcal{K}}_\Delta(n)$ of $\mathcal{K}_\Delta(n)$. This is finally used to find a monomorphism from $U_\Delta(n)$ into $\hat{\mathcal{K}}_\Delta(n)$.  

It should be pointed out that the approach in the paper under review is completely algebraic contrary to the mostly geometric approaches developed previously. The modified approach of the authors also works for $U(n)$ and yields a realization of $U(n)$ without using the stabilization property leading to the limit algebra $K(n)$. Moreover, it is proved that the explicit algebra homomorphism from $U_\Delta(n)$ to $S_\Delta(n, r)$ used by the authors coincides with the one obtained from the natural $U_\Delta(n)$-action on the corresponding tensor space.