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Self-dual and quasi self-dual algebras.

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In general, Hochschild cohomology of an associative algebra $A$ with coefficients in itself is not functorial. The aim of the paper is to find conditions on the algebra to ensure this functoriality. The author says that an associative algebra $A$ is quasi self-dual if there is an isomorphism $H^∗(A, A) \rightarrow H^∗(A, A^{\text{\text{op}}})$ of graded vector spaces, where $A^{\text{\text{op}}}$ denotes the linear dual of $A$ obtained from the bimodule action for the opposite algebra $A^{\text{\text{op}}}$ of $A$ by interchanging the left and right actions. An associative algebra $A$ is called self-dual if $A$ and $A^{\text{\text{op}}}$ are isomorphic as $A$-bimodules.

It is proved that for any quasi self-dual algebra $A$ the Hochschild cohomology $H^∗(A, A)$ is a contravariant functor of $A$ (or more precisely, in the category of pairs $(A, \eta)$ of quasi self-dual algebras $A$ and isomorphisms $H^∗(A, A) \rightarrow H^∗(A, A^{\text{\text{op}}})$ of graded vector spaces). Moreover, a finite-dimensional algebra is self-dual precisely when it is a symmetric Frobenius algebra. Finally, the author shows that simplicial cohomology is the same as the Hochschild cohomology $H^∗(A, A)$ of some poset algebra $A$, and poset algebras are quasi self-dual.