In several papers Kashiwara developed a theory of crystal bases as a tool for investigating the structure of integrable modules over the quantized universal enveloping algebra $U_q(\mathfrak{g})$ of a Kac-Moody algebra $\mathfrak{g}$. Let now $\mathfrak{g}$ be any symmetrizable Kac-Moody algebra, let $X$ denote the weight lattice of $\mathfrak{g}$, and set $X_\mathbb{R} := \mathbb{R} \otimes_\mathbb{Z} X$. In this case Littelmann [Invent. Math. 116 (1994), no. 1-3, 329–346; MR1253196 (95f:17023) and Ann. of Math. (2) 142 (1995), no. 3, 499–525; MR1356780 (96m:17011)] considers the set $\Pi$ of all equivalence classes of continuous piecewise linear paths $\pi : [0, 1] \to X_\mathbb{R}$ with $\pi(0) = 0$ modulo reparametrization. Moreover, for every simple root $\alpha$ Littelmann introduces an action of $e_\alpha$ and $f_\alpha$ (the so-called root operators) on $\Pi \cup \{0\}$ induced by the reflection through the hyperplane perpendicular to $\alpha$ and having properties very similar to the analogous operators considered by Kashiwara. Let $\lambda$ be a dominant integral weight and let $\pi$ be any path with $\pi(1) = \lambda$ that is contained in the fundamental Weyl chamber. Then the highest weight crystal $B(\lambda)$ associated to $\lambda$ is realized by the orbit of the equivalence class of $\pi$ under the action of the root operators.

In the paper under review the authors generalize Littlemann’s path model for a highest weight crystal $B(\lambda)$ by allowing parametrizations of paths on an interval $[0, c]$, where $c$ is any positive number or $+\infty$ and defining modified root operators $\tilde{e}_\alpha$ and $\tilde{f}_\alpha$ acting directly on the set of such paths with a fixed parametrization. This action of the modified root operators induces an action of the root operators $e_\alpha$ and $f_\alpha$ on the set of equivalence classes of paths. Let $\rho$ denote the sum of the fundamental weights of $\mathfrak{g}$. If $c \neq +\infty$, then Littlemann’s path model is recovered, and in the case $c = +\infty$ the orbit of the equivalence class of the path $\pi_{+\infty} : [0, +\infty) \to X_\mathbb{R}$, $t \mapsto t\rho$ under the action of the root operators is isomorphic to the crystal basis $B(\infty)$ of the negative part of $U_q(\mathfrak{g})$. The authors also conjecture for $\mathfrak{g}$ being of affine or indefinite type that the same holds true if the equivalence class of the path $\pi$ is replaced by the path $\pi$ itself and the root operators $e_\alpha$ and $f_\alpha$ are replaced by the modified root operators $\tilde{e}_\alpha$ and $\tilde{f}_\alpha$. This conjecture is proved in the case that $\mathfrak{g}$ is of finite type.