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Jantzen filtration and strong linkage principle for modular Lie superalgebras.

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Let $\mathfrak{g}$ be the analogue of a basic classical complex Lie superalgebra over an algebraically closed field $\mathbb{F}$ of characteristic $p > 2$. Then the even part of $\mathfrak{g}$ is the Lie algebra of a reductive algebraic group defined over $\mathbb{F}$. In particular, $\mathfrak{g}$ is a restricted Lie superalgebra. Moreover, for large enough $p$ the Lie superalgebra $\mathfrak{g}$ admits a non-degenerate even supersymmetric invariant bilinear form. More precisely, there exists an algebraic supergroup $G$ with Lie superalgebra $\mathfrak{g}$ such that $G$ has a subgroup scheme $G_{\text{ev}}$, which is a connected reductive algebraic group with Lie algebra $\mathfrak{g}_{\text{ev}}$, the even part of $\mathfrak{g}$, and there is a well-defined action of $G_{\text{ev}}$ on $\mathfrak{g}$ that restricts to the adjoint action on $\mathfrak{g}_{\text{ev}}$.

In the paper under review the authors consider the category of all finite-dimensional restricted $\mathfrak{g}$-modules $\mathcal{M}$. By fixing a maximal torus $\Xi$ of $G_{\text{ev}}$ many properties of $\mathcal{M}$ can be obtained from the full subcategory $\mathcal{C}$ that consists of those objects in $\mathcal{M}$ that admit a rational $\Xi$-action which is compatible with the $\mathfrak{g}$-action. This strategy was successfully applied by J. C. Jantzen to Weyl modules over Frobenius kernels of semisimple algebraic groups [J. Reine Angew. Math. **317** (1980), 157–199; MR581341 (82b:20057)] and to restricted as well as non-restricted baby Verma modules over Lie algebras of reductive algebraic groups [J. Pure Appl. Algebra **152** (2000), no. 1-3, 133–185; MR1783993 (2001j:17016)]. The main results of the paper under review are an super analogue of the Jantzen filtration for baby Verma modules and a corresponding Jantzen sum formula in the Grothendieck group of $\mathcal{C}$. As a consequence the authors obtain a strong linkage principle which gives a necessary condition for possible compositions factors of baby Verma modules in $\mathcal{C}$.