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Restricted infinitesimal deformations of restricted simple Lie algebras.


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In this paper the author computes the restricted infinitesimal deformations of the restricted simple Lie algebras of non-classical type over ground fields of characteristic $p > 3$. In three previous papers [J. Algebra 320 (2008), no. 12, 4102–4131; MR2457812 (2009k:17036); J. Pure Appl. Algebra 213 (2009), no. 9, 1702–1721; MR2518170 (2010h:17023); Comm. Algebra 37 (2009), no. 6, 1850–1872; MR2530748 (2010g:17024)] the author already determined the ordinary infinitesimal deformations of these Lie algebras. For the Jacobson-Witt algebras, most of the restricted contact Lie algebras (namely, those for which the toral rank +2 is not divisible by the characteristic of the ground field), and the restricted Melikian algebra, restricted and ordinary infinitesimal deformations coincide since these Lie algebras have only inner derivations, and therefore the corresponding results immediately follow from the previous papers of the author mentioned above. If the characteristic of the ground field does divide the toral rank +2, the same holds true for the restricted contact Lie algebras, but because of the existence of outer derivations this case requires a little more work. Only for the restricted special Lie algebras and the restricted Hamiltonian algebras (where the latter is as usual the most complicated case) the space of restricted infinitesimal deformations is a proper subspace of the space of ordinary infinitesimal deformations.

Over 40 years ago Rudakov [Izv. Akad. Nauk SSSR Ser. Mat. 35 (1971), 1113–1119 (in Russian); MR0291235 (45 #329)] proved that simple Lie algebras of classical type are rigid if the characteristic of the ground field is at least five. Contrary to this, restricted simple Lie algebras of Cartan type and the restricted Melikian algebra are never rigid. The results of the paper under review in conjunction with Rudakov’s result and the recently completed Block-Wilson-Strade-Premet classification of restricted simple Lie algebras over an algebraically closed field of characteristic $p > 3$ gives a complete understanding of the restricted infinitesimal deformations of restricted simple Lie algebras in characteristic larger than three.