
Mathematics Subject Classification 2000: *17B37, 14A22, 14F05, 14M15, 18E30, 18F20

Keywords: Complex semisimple group; quantum group at a root of unity; small quantum group; triangulated category; bounded derived category; equivariant coherent sheaf; nilpotent cone; Springer resolution; principal block; Hochschild cohomology; center

Reviewer: Jörg Feldvoss (8086)

Let $G$ be a complex connected semisimple group of adjoint type with Lie algebra $\mathfrak{g}$, root system $R$, and root lattice $\mathbb{Y}$. Fix a Borel subgroup $B$ of $G$ with Lie algebra $\mathfrak{b}$ and nilradical $\mathfrak{n} \subset \mathfrak{b}$. Let $l$ be an odd integer which is larger than the Coxeter number of $R$ and relatively prime to its index of connection (and not divisible by 3 if $R$ has a component of type $G_2$) and choose a primitive $l$-th root of unity $\xi$. S. Arkhipov, the first author of the paper under review, and V. Ginzburg [J. Am. Math. Soc. 17, No. 3, 595-678 (2004; Zbl. 1061.17013)] have established the existence of a triangulated functor $F$ from the bounded derived category $D^b\text{Coh} (\tilde{N})$ of $G \times \mathbb{C}^*$-equivariant coherent sheaves on the Springer resolution $\tilde{N} = G \times_B \mathfrak{n}$ of the singularities of the nilpotent cone $N$ of $\mathfrak{g}$ to the bounded derived category $D^b\text{block}(U)$ of finite-dimensional $\mathbb{Y}$-graded modules in the principal block of Lusztig’s quantum group $U$ for $\mathfrak{g}$ at the root of unity $\xi$ satisfying certain desirable properties (e.g., the image of $F$ generates $D^b\text{block}(U)$ as a triangulated category). The main result of the paper under review is to prove the existence of a triangulated functor $F_u$ from the bounded derived category $D^b\text{Coh}^{G \times \mathbb{C}^*} (\tilde{N})$ of $\mathbb{C}^*$-equivariant coherent sheaves on the Springer resolution $\tilde{N}$ to the bounded derived category $D^b\text{block}(u)$ of finite-dimensional modules in the principal block of the small quantum group $u$ associated to $U$ which has similar properties as $F$ and is compatible with $F$ (i.e., the composition of $F_u$ and the forgetful functor from $D^b\text{Coh}^{G \times \mathbb{C}^*} (\tilde{N})$ to $D^b\text{Coh}^{\mathbb{C}^*} (\tilde{N})$ coincides with the composition of the restriction functor from $D^b\text{block}(U)$ to $D^b\text{block}(u)$ and $F$). As an application the authors obtain a geometric description of the Hochschild cohomology of the principal block of the small quantum group as the total cohomology of $\tilde{N}$ with coefficients in the coherent sheaf of poly-vector fields on $\tilde{N}$. Since the center is just Hochschild cohomology in degree zero, this then can be used to give an explicit description of a certain canonically defined subalgebra of the center of $u$ which was obtained previously by the second author under more restrictive assumptions [J. Algebra 262, No. 2, 313-331 (2003; Zbl. 1049.17011)].