As a geometric counterpart of the $R$-matrix of a quantum group, A. Weinstein and P. Xu [Commun. Math. Phys. 148, No. 2, 309-343 (1992; Zbl. 0849.17015)] introduce a certain Lagrangian submanifold $\mathcal{R}$ in the cartesian square of the symplectic groupoid associated to a Poisson-Lie group $G$. For every symplectic leaf $S$ in $G$ it is shown that $\mathcal{R}$ induces a symplectic automorphism of $S \times S$ which satisfies the set-theoretic quantum Yang-Baxter equation and yields a braiding on the function algebra of $S$. Since $G$ is the union of its symplectic leaves, one also obtains a braiding on the function algebra of $G$. Recently, the authors of the paper under review, alone and jointly with F. Gavarini, have related this explicitly to the theory of quantum groups as follows. Let $(\mathfrak{g}, r)$ be a finite-dimensional quasi-triangular Lie bialgebra and let $(U_\hbar(\mathfrak{g}), R)$ be the quasi-triangular quantized universal enveloping algebra with classical limit $(\mathfrak{g}, r)$. Then one can associate a quantized formal series algebra (QFSA) $U_\hbar(\mathfrak{g})' \subset U_\hbar(\mathfrak{g})$ to $(U_\hbar(\mathfrak{g}), R)$ which is a flat deformation of the function algebra of the group $G^*$ with Lie algebra $\mathfrak{g}^*$. Moreover, one can prove that the completed tensor square of $U_\hbar(\mathfrak{g})'$ is invariant under the adjoint action Ad($R$) of $R$ and that Ad($R|_{\hbar=0}$) coincides with the action induced by $\mathcal{R}$ on the completed tensor square of the function algebra of $G^*$. In the paper under review the authors study the analogues of these results for quasi-quantum groups.

Let $\mathbb{K}$ be a field of characteristic zero and let $U$ be a quasi-Hopf quantized universal enveloping algebra (QHQUE algebra) over $\mathbb{K}$, i.e., $U$ is a quasi-Hopf algebra which is topologically free as a $\mathbb{K}[\hbar]$-module and the reduction $U/\hbar U$ of $U$ modulo $\hbar$ is the universal enveloping algebra of the Lie algebra $\mathfrak{g}$ of its primitive elements. The authors define a QFSA $U' \subset U$ corresponding to $U$, and show that the factor ring $U'/\hbar U'$ is a complete local commutative ring. Furthermore, it is proved under a certain $\hbar$-adic valuation condition that $U'/\hbar U'$ is as a filtered algebra isomorphic to the formal series completion of the symmetric algebra of $\mathfrak{g}$. QHQUE algebras satisfying this $\hbar$-adic valuation condition are called admissible and admissible twists are defined by a similar condition. Then it is shown that admissible QHQUE algebras twisted by admissible twists are again admissible with the same QFSA and that any QHQUE algebra can be twisted into an admissible QHQUE algebra. The paper concludes by proving the related result that any associator can be twisted into a unique Lie associator.