The purpose of the paper under review is to report on the recent progress in the representation theory of Lie algebras of reductive algebraic groups over an algebraically closed field of prime characteristic and thereby to update the earlier surveys by the author [Representation theories and algebraic geometry. Proceedings of the NATO Advanced Study Institute, Montreal, Canada, July 28-August 8, 1997. Broer, A. (ed.) et al., Dordrecht: Kluwer Academic Publishers. NATO ASI Ser., Ser. C, Math. Phys. Sci. 514, 185-235 (1998; Zbl. 0974.17022)] and by J. E. Humphreys [Bull. Amer. Math. Soc. (N.S.) 35, No. 2, 105-122 (1998; Zbl. 0962.17013)]. The author begins in section A by reminding the reader of some basic facts from the representation theory of restricted Lie algebras. In this section the author proves the well-known fact that the dimensions of the simple modules of a finite-dimensional (restricted) Lie algebra are bounded as well as Zassenhaus’ result saying that the maximum of these dimensions is always a power of the characteristic of the underlying field.

From the next section on the author restricts himself to Lie algebras of connected reductive algebraic groups. In section B he proves that every simple module of such a Lie algebra is the homomorphic image of a baby Verma module. Then three standard conditions on the group or the characteristic $p$ of the ground field are introduced which will be assumed to be satisfied for most of the rest of the paper. In particular, this
enables the author to use Premet’s theorem which gives a lower bound (depending on \( \chi \)) for the \( p \)-powers dividing the dimension of every simple module with \( p \)-character \( \chi \). Since it is enough to consider the case of a nilpotent \( p \)-character, only these are considered in the remaining parts of the paper. Section C contains the modular analogue of the Harish-Chandra isomorphism and the block decomposition of the reduced universal enveloping algebra. In Section D the author discusses the special case of \( p \)-characters in standard Levi form which includes all nilpotent \( p \)-characters in types \( A \), \( B_2 \), and \( B_3 \). In this case every baby Verma module has a unique maximal submodule which leads to a classification of the simple modules as in the restricted case. There are also graded versions of the category of all modules with a fixed \( p \)-character in standard Levi form and a conjecture of Lusztig predicting the multiplicity of a simple module in a baby Verma module as the value of a certain periodic polynomial at \(-1\). The author includes a list of the cases in which he confirmed this conjecture by using his sum formula in conjunction with Premet’s theorem, translation functors, and projective indecomposable modules. The author concludes this section by proving that the conjecture of Lusztig yields a finite character formula for a simple module with a \( p \)-character in standard Levi form provided another conjecture of Lusztig about the periodic polynomials holds true.

Section E is devoted to the geometric construction of representations of Lie algebras of reductive algebraic groups and especially the recent results of Bezrukavnikov, Mirković, and Rumynin. Their main result is a derived equivalence between the category of finitely generated modules which are annihilated by the kernel of the principal central character and the coherent sheaves supported on the formal neighborhood of the corresponding generalized Springer fiber. In particular, this confirms a conjecture of Lusztig predicting the number of isomorphism classes of simple modules in the “principal” block. In Section F the author supplements his results for subregular nilpotent \( p \)-characters from [Represent. Theory 3, No. 8, 153-222 (1999; Zbl. 0998.17003)] in the non-simply laced cases by using the results of Bezrukavnikov, Mirković, and Rumynin. In the last section G the author studies the projective indecomposable modules and especially a conjecture of Lusztig on the Cartan matrix of a block of a reduced universal enveloping algebra. Lusztig has shown that his conjecture describing the composition factors of the restricted baby Verma modules implies the conjecture on the Cartan matrix of a block of the restricted universal enveloping algebra. The author concludes this survey by proving that the converse statement is also true.