Let $L$ be a finite-dimensional Lie algebra over a field of positive characteristic. Motivated by the representation theory of Lie algebras of reductive groups the authors consider bases of a special form for those primitive factor algebras of the universal enveloping algebra $\mathcal{U}(L)$ of $L$ that correspond to simple $L$-modules of maximal dimension.